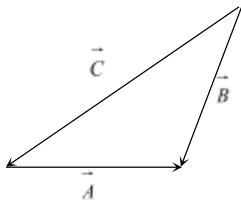


SECTION - A

- The expression $\left(\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}\right)$ is a
 - Unit vector
 - Null vector
 - Vector of magnitude $\sqrt{2}$
 - Scalar
- The angle made by the vector $A = \hat{i} + \hat{j}$ with x-axis is
 - 90°
 - 45°
 - 22.5°
 - 30°
- If a unit vector is represented by $0.5\hat{i} + 0.8\hat{j} + c\hat{k}$, then the value of 'c' is
 - 1
 - $\sqrt{0.11}$
 - $\sqrt{0.01}$
 - $\sqrt{0.39}$
- Find the value of $\sin 105^\circ$
 - $\frac{\sqrt{6} + \sqrt{2}}{4}$
 - $\frac{1 + \sqrt{3}}{2\sqrt{2}}$
 - (1) & (2)
 - None
- The angle between the two vectors $\vec{A} = 3\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{B} = 3\hat{i} + 4\hat{j} + 5\hat{k}$ is
 - 60°
 - Zero
 - 90°
 - None of these
- Find maximum value of 'y' where $y = 2 \sin \theta + \sqrt{5} \cos \theta$.
 - 3
 - $2 + \sqrt{5}$
 - $2\sqrt{5}$
 - $\sqrt{5}$
- For the figure



- $\vec{A} + \vec{B} = \vec{C}$
 - $\vec{B} + \vec{C} = \vec{A}$
 - $\vec{C} + \vec{A} = \vec{B}$
 - $\vec{A} + \vec{B} + \vec{C} = 0$
- Let $\vec{C} = \vec{A} + \vec{B}$ then
 - $|\vec{C}|$ is always greater than $|\vec{A}|$
 - It is possible to have $|\vec{C}| < |\vec{A}|$ and $|\vec{C}| < |\vec{B}|$
 - C is always equal to A + B
 - C is never equal to A + B

- If a vector $2\hat{i} + 3\hat{j} + 8\hat{k}$ is perpendicular to the vector $4\hat{j} - 4\hat{i} + \alpha\hat{k}$. Then the value of α is
 - 1
 - $\frac{1}{2}$
 - $-\frac{1}{2}$
 - 1
- If $\vec{A} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{B} = 2\hat{i} - 2\hat{j} + 4\hat{k}$ then value of $|\vec{A} \times \vec{B}|$ will be
 - $8\sqrt{2}$
 - $8\sqrt{3}$
 - $8\sqrt{5}$
 - $5\sqrt{8}$
- If $\vec{P} \cdot \vec{Q} = PQ$, then angle between $\vec{P}$ and $\vec{Q}$ is
 - 0°
 - 30°
 - 45°
 - 60°
- Correct value of $\cos(2^\circ)$
 - 2°
 - $\frac{\pi}{50}$
 - 1
 - 0
- If $KE = \frac{P^2}{2m}$ then draw graph between KE and P.
 -
 -
 -
 -
- A body is attached to a spring whose other end is fixed. If the spring is elongated by x, its potential energy is $U = 20x^2$, where x is in metre and U is in joule. U-x graph is
 -
 -
 -
 -
- The angles which a vector $\hat{i} + \hat{j} + \sqrt{2}\hat{k}$ makes with X, Y and Z axes respectively are
 - $60^\circ, 60^\circ, 60^\circ$
 - $45^\circ, 45^\circ, 45^\circ$
 - $60^\circ, 60^\circ, 45^\circ$
 - $45^\circ, 45^\circ, 60^\circ$

16. If $\vec{A} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{B} = -\hat{i} + 3\hat{j} + 4\hat{k}$ then projection of $\vec{A}$ on $\vec{B}$ will be

- (1) $\frac{3}{\sqrt{13}}$ (2) $\frac{3}{\sqrt{26}}$
(3) $\sqrt{\frac{3}{26}}$ (4) $\sqrt{\frac{3}{13}}$

17. Which of the following is the unit vector perpendicular to $\vec{A}$ and $\vec{B}$

- (1) $\frac{\hat{A} \times \hat{B}}{AB \sin \theta}$ (2) $\frac{\hat{A} \times \hat{B}}{AB \cos \theta}$
(3) $\frac{\vec{A} \times \vec{B}}{AB \sin \theta}$ (4) $\frac{\vec{A} \times \vec{B}}{AB \cos \theta}$

18. Two vectors $P = 2\hat{i} + b\hat{j} + 2\hat{k}$ and $Q = \hat{i} + \hat{j} + \hat{k}$ will be parallel if

- (1) $b = 0$ (2) $b = 1$
(3) $b = 2$ (4) $b = -4$

19. $\frac{d}{dx}(\sqrt{x} + x^2 + 10)$

- (1) $\frac{1}{2x} + 2x$ (2) $\frac{1}{2\sqrt{x}} + 2x$
(3) $\frac{1}{2\sqrt{x}} + 2x + 1$ (4) $\frac{1}{\sqrt{x}} + 2x$

20. If $y = x^2 + 4x^3 - 8x + 4$, then find $\frac{dy}{dx}$

- (1) $2x + 4x^2 - x$ (2) $2x + 12x^2 - 8$
(3) $2x + 4x^3 - 8$ (4) $2x + 12x^2 - x$

21. The minimum value of $y = 5x^2 - 2x + 1$ is

- (1) $1/5$ (2) $2/5$
(3) $4/5$ (4) $3/5$

22. $\int (x^3 + x + 2) dx = ?$

- (1) $\frac{x^4}{4} + \frac{x^2}{2} + 2x + c$ (2) $\frac{x^2}{2} + \frac{x}{2} + 2x + c$
(3) $\frac{x^3}{3} + \frac{x^2}{2} + 2 + c$ (4) None

23. $\vec{A}$ and $\vec{B}$ are two vectors and θ is the angle between them, if $|\vec{A} \times \vec{B}| = \sqrt{3}(\vec{A} \cdot \vec{B})$, the value of θ is

- (1) 90° (2) 60°
(3) 45° (4) 30°

24. If $\sin \theta = \frac{4}{5}$ then $\tan \theta$ will be

- (1) $\frac{4}{9}$ (2) $\frac{3}{4}$
(3) $\frac{4}{5}$ (4) $\frac{4}{3}$

25. $\int \left(\cos \frac{x}{2} - \sin \frac{x}{2} \right)^2 dx =$

- (1) $x + \cos x + c$ (2) $2\cos^2 \frac{x}{2} + c$
(3) $\frac{1}{3} \left(\cos \frac{x}{2} - \frac{x}{2} \right)^3 + c$ (4) $x - \cos x + c$

26. $\frac{d}{dx} \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right)^2 =$

- (1) $1 - \frac{1}{x^2}$ (2) $1 + \frac{1}{x^2}$
(3) $1 - \frac{1}{2x}$ (4) None of these

27. If $y = x^2$, then $\int_0^1 y dx$ will be:

- (1) $\frac{x^3}{3} + C$ (2) $1/3$
(3) $2/3$ (4) 0

28. Value of $\int_0^{\pi/2} \cos 3t dt$ is

- (1) $\frac{2}{3}$ (2) $-\frac{1}{3}$
(3) $-\frac{2}{3}$ (4) $\frac{1}{3}$

29. Can the resultant of 2 vectors be zero

- (1) Yes, when the 2 vectors are same in magnitude and direction
(2) No
(3) Yes, when the 2 vectors are same in magnitude but opposite in sense
(4) Yes, when the 2 vectors are same in magnitude making an angle of $\frac{2\pi}{3}$ with each other

30. Two equals forces are acting at a point with an angle of 60° between them. If the resultant force is equal to $40\sqrt{3}$ N, the magnitude of each force is

- (1) 40 N (2) 20 N
(3) 80 N (4) 30 N

31. $\sin(90^\circ - \theta)$ is equal to:

- (1) $\sin \theta$ (2) $-\sin \theta$
(3) $\cos \theta$ (4) $-\cos \theta$

32. Given $x^2 + 7x + 12 = 0$, find the root of x

- (1) $x = \frac{3}{2}, -4$ (2) $x = -3, -4$
(3) $x = \frac{3}{2}, 4$ (4) $x = \frac{3}{2}, -2$

33. If $y = e^{-ax}$, then find double differentiation of y .

- (1) αe^{-ax} (2) $-\alpha e^{-ax}$
 (3) e^{-ax} (4) $\alpha^2 e^{-ax}$

34. $\int e^{6x} dx$

- (1) $e^{5x} + C$ (2) $e^{5x} \cdot \frac{5x^2}{2} + C$
 (3) $\frac{e^{6x}}{6} + C$ (4) $e^x + C$

35. The sum of the series $1 + 1/4 + 1/16 + 1/64 + \dots \infty$ is

- (1) $8/7$ (2) $6/5$
 (3) $5/4$ (4) $4/3$

SECTION-B

36. Find the value of $\sin 15^\circ$.

- (1) $\frac{\sqrt{3}-1}{\sqrt{2}}$ (2) $\frac{\sqrt{3}-\sqrt{2}}{2\sqrt{2}}$
 (3) $\frac{\sqrt{3}-1}{2\sqrt{2}}$ (4) $\frac{1}{2}$

37. Convert angle from radian to degree $\frac{\pi}{3}$ rad:

- (1) 60° (2) 30°
 (3) 45° (4) 0°

38. Find the value of $\log_{10} 10^{35}$

- (1) 28 (2) 32
 (3) 36 (4) 35

39. What is the value of $\log_2 16$?

- (1) 8 (2) 4
 (3) $1/8$ (4) 16

40. The slope of straight line $\sqrt{3}y = 3x + 4$ is

- (1) 3 (2) $\sqrt{3}$
 (3) $\frac{1}{\sqrt{3}}$ (4) $\frac{1}{3}$

41. Which of the following is correct for $(64)^{2/3}$

- (1) 16 (2) 32
 (3) 4 (4) 8

42. The angle made by the vector $\vec{A} = \hat{i} + \hat{j}$ with x-axis is

- (1) 90° (2) 45°
 (3) 22.5° (4) 30°

43. Angular momentum is

- (1) A scalar (2) A polar vector
 (3) An axial vector (4) None of these

44. If $\vec{A} \times \vec{B} = \vec{B} \times \vec{A}$, then the angle between $\vec{A}$ and $\vec{B}$ is

- (1) $\frac{\pi}{2}$ (2) $\frac{\pi}{3}$
 (3) π (4) $\frac{\pi}{4}$

45. If a vector $\vec{P}$ making angles α, β and γ respectively with X, Y and Z axes respectively.

Then $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma =$

- (1) 0 (2) 1
 (3) 2 (4) 3

46. Forces 7 N, 24 N, 25 N act at a point in mutually perpendicular directions. The magnitude of the resultant force is :

- (1) 19 N (2) 13 N
 (3) 26 N (4) $25\sqrt{2}$ N

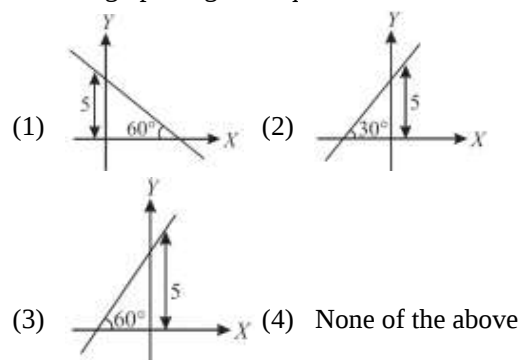
47. The angle between the vectors $\vec{A}$ and $\vec{B}$ is θ . The value of the triple product $\vec{A} \cdot (\vec{B} \times \vec{A})$ is

- (1) $A^2 B$ (2) Zero
 (3) $A^2 B \sin \theta$ (4) $A^2 B \cos \theta$

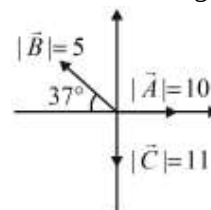
48. Given that, $\sin A = \frac{1}{2}$ and $\cos B = \frac{1}{\sqrt{2}}$ then value of $(A+B)$ will be:

- (1) 30° (2) 45°
 (3) 75° (4) 15°

49. Plot the graph of given equation, $Y = \sqrt{3}X + 5$



50. Find the resultant of following vectors



- (1) 8 (2) 6
 (3) 10 (4) 20

Solution

1. (1)

$$\vec{P} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j} \therefore |\vec{P}| = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = 1$$

$\therefore$ It is a unit vector.

2. (2)

$$\vec{A} = \hat{i} + \hat{j} \Rightarrow |\vec{A}| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$\cos \alpha = \frac{A_x}{|\vec{A}|} = \frac{1}{\sqrt{2}} = \cos 45^\circ \therefore$$

3. (2)

Magnitude of unit vector = 1

$$\Rightarrow \sqrt{(0.5)^2 + (0.8)^2 + c^2} = 1$$

By solving we get $c = \sqrt{0.11}$

4. (3)

$$\sin(105) = \sin(60 + 45^\circ)$$

$$= \sin 60 \cos 45 + \cos 60 \sin 45^\circ$$

$$= \frac{\sqrt{6} + \sqrt{2}}{4} = \frac{1 + \sqrt{3}}{2\sqrt{2}}$$

5. (2)

$$\cos \theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|} = \frac{9 + 16 + 25}{\sqrt{9 + 16 + 25} \sqrt{9 + 16 + 25}} = \frac{50}{50} = 1$$

$$\Rightarrow \cos \theta = 1 \therefore \theta = \cos^{-1}(1) = \text{zero}$$

6. (1)

$$-\sqrt{(2)^2 + (\sqrt{5})^2} < 2 \sin \theta + \sqrt{5} \cos \theta < \sqrt{(2)^2 + (\sqrt{5})^2}$$

$$-3 < 2 \sin \theta + \sqrt{5} \cos \theta < 3$$

then maximum value = 3

7. (3)

By law of triangle (graphical method)

$$\vec{C} + \vec{A} = \vec{B}$$

8. (2)

$$\vec{C} + \vec{A} = \vec{B}$$

The value of C lies between $A - B$ and $A + B$

$$\therefore |\vec{C}| < |\vec{A}| \text{ or } |\vec{C}| < |\vec{B}|$$

9. (3)

Given vectors can be rewritten as

$$\vec{A} = 2\hat{i} + 3\hat{j} + 8\hat{k} \text{ and } \vec{B} = -4\hat{i} + 4\hat{j} + \alpha\hat{k}$$

Dot product of these vectors should be equal to zero because they are perpendicular.

$$\therefore \vec{A} \cdot \vec{B} = -8 + 12 + 8\alpha = 0 \Rightarrow 8\alpha = -4$$

$$\Rightarrow \alpha = -1/2$$

10. (2)

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 2 \\ 2 & -2 & 4 \end{vmatrix}$$

$$= (1 \times 4 - 2 \times -2)\hat{i} + (2 \times 2 - 4 \times 3)\hat{j} + (3 \times -2 - 1 \times 2)\hat{k}$$

$$= 8\hat{i} - 8\hat{j} - 8\hat{k}$$

$$\therefore \text{Magnitude of } \vec{A} \times \vec{B} = |\vec{A} \times \vec{B}|$$

$$= \sqrt{(8)^2 + (-8)^2 + (-8)^2} = 8\sqrt{3}$$

11. (1)

$$\cos \theta = \frac{\vec{P} \cdot \vec{Q}}{PQ} = 1 \therefore \theta = 0^\circ$$

12. (3)

$$\cos 2^\circ = 1$$

Because angle is very small.

13. (2)

$$KE \propto P^2 \quad (\text{Parabolic graph})$$

14. (1)

$$U \propto x^2 \quad (\text{Parabolic graph})$$

15. (3)

$$\vec{R} = \hat{i} + \hat{j} + \sqrt{2}\hat{k}$$

Comparing the given vector with

$$\vec{R} = R_x\hat{i} + R_y\hat{j} + R_z\hat{k}$$

$$R_x = 1, R_y = 1, R_z = \sqrt{2} \text{ and}$$

$$|\vec{R}| = \sqrt{R_x^2 + R_y^2 + R_z^2} = 2$$

$$\cos \alpha = \frac{R_x}{R} = \frac{1}{2} \Rightarrow \alpha = 60^\circ$$

$$\cos \beta = \frac{R_y}{R} = \frac{1}{2} \Rightarrow \beta = 60^\circ$$

$$\cos \gamma = \frac{R_z}{R} = \frac{1}{\sqrt{2}} \Rightarrow \gamma = 45^\circ$$

16. (2)

$$|\vec{A}| = \sqrt{2^2 + 3^2 + (-1)^2} = \sqrt{4 + 9 + 1} = \sqrt{14}$$

$$|\vec{B}| = \sqrt{(-1)^2 + 3^2 + 4^2} = \sqrt{1 + 9 + 16} = \sqrt{26}$$

$$\vec{A} \cdot \vec{B} = 2(-1) + 3 \times 3 + (-1)(4) = 3$$

$$\text{The projection of } \vec{A} \text{ on } \vec{B} = \frac{\vec{A} \cdot \vec{B}}{|\vec{B}|} = \frac{3}{\sqrt{26}}$$

17. (3)

Vector perpendicular to A and B,

$$\vec{A} \times \vec{B} = AB \sin \theta \hat{n}$$

$\therefore$ Unit vector perpendicular to A and B

$$\hat{n} = \frac{\vec{A} \times \vec{B}}{|\vec{A}| |\vec{B}| \sin \theta}$$

18. (3)

P and Q will be parallel if $\frac{2}{1} = \frac{b}{1} = \frac{2}{1} \therefore b = 2$

19. (2)

$$\begin{aligned} \frac{d}{dx}(x^{1/2} + x^2 + 10) \\ = \frac{1}{2}x^{-1/2} + 2x + 0 \\ = \frac{1}{2\sqrt{x}} + 2x \end{aligned}$$

20. (2)

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx}(x^2 + 4x^3 - 8x + 4) \\ &= 2x + 12x^2 - 8 \end{aligned}$$

21. (3)

$$y = 5x^2 - 2x + 1$$

$$\frac{dy}{dx} = 10x - 2$$

$$\text{For min}^m, \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = 10x - 2 = 0$$

$$\boxed{x = \frac{1}{5}}$$

And

$$\frac{d^2y}{dx^2} = 10 > 0$$

then

$$y_{\min} = 5\left(\frac{1}{5}\right)^2 - 2 \times \frac{1}{5} + 1 = \frac{4}{5}$$

22. (1)

$$\begin{aligned} \int (x^3 + x + 2) dx \\ = \frac{x^4}{4} + \frac{x^2}{2} + 2x + C \end{aligned}$$

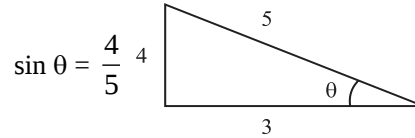
23. (2)

$$|\vec{A} \times \vec{B}| = \sqrt{3} \vec{A} \vec{B}$$

$$AB \sin \theta = \sqrt{3} AB \cos \theta$$

$$\begin{aligned} \tan \theta &= \sqrt{3} \\ \theta &= 60^\circ \end{aligned}$$

24. (4)



$$\boxed{\tan \theta = \frac{4}{3}}$$

25. (1)

$$\begin{aligned} \int \left(\cos \frac{x}{2} - \sin \frac{x}{2} \right)^2 dx \\ = \int \left\{ \cos^2 \frac{x}{2} + \sin^2 \frac{x}{2} - 2 \sin \frac{x}{2} \cos \frac{x}{2} \right\} dx \\ = \int (1 - \sin x) dx = x + \cos x + c \end{aligned}$$

26. (1)

$$\frac{d}{dx} \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right)^2 = \frac{d}{dx} \left[x + \frac{1}{x} + 2 \right] = 1 - \frac{1}{x^2}$$

27. (2)

$$\int_0^1 y dx = \int_0^1 x^2 dx = \left[\frac{x^3}{3} \right]_0^1 = \frac{1}{3}$$

28. (2)

$$\begin{aligned} \int_0^{\pi/2} \cos 3t dt &= \left[\frac{\sin 3t}{3} \right]_0^{\pi/2} \\ &= \frac{1}{3} [-1 - 0] = -\frac{1}{3} \end{aligned}$$

29. (3)

30. (1)

$$\begin{aligned} 2F \cos \left(\frac{60^\circ}{2} \right) &= 40\sqrt{3} \\ \Rightarrow 2F \frac{\sqrt{3}}{2} &= 40\sqrt{3} \\ \Rightarrow \boxed{F = 40 \text{ N}} \end{aligned}$$

31. (3)

$$\sin (90 - \theta) = + \cos \theta$$

32. (2)

$$\begin{aligned} x^2 + 7x + 12 &= 0 \\ (x + 3)(x + 4) &= 0 \\ x &= -3, -4 \end{aligned}$$

33. (4)

$$y = e^{-\alpha x}$$

$$\frac{dy}{dx} = -\alpha e^{-\alpha x}$$

$$\boxed{\frac{d^2 y}{dx^2} = +\alpha^2 e^{-\alpha x}}$$

$$= \alpha^2 e^{-\alpha x}$$

34. (3)

35. (4)

$$S_{\infty} = \frac{a}{1-r} = \frac{1}{1-1/4} = \frac{4}{3}$$

36. (3)

$$\sin 15^\circ = \sin(45^\circ - 30^\circ)$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$= \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ$$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2} = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

37. (1)

$$\frac{\pi}{3} \times \frac{180}{\pi} = 60^\circ$$

38. (4)

$$\log a^b = b \log_a a$$

$$\log_{10}^{10} = 1$$

39. (2)

$$\log_2 16 = \log_2 2^4 = 4 \log_2 2 = 4$$

40. (2)

$$\sqrt{3}y = 3x + 4$$

$$y = \sqrt{3}x + \frac{4}{\sqrt{3}}$$

$$\text{slope} = \sqrt{3}$$

41. (1)

Fact

42. (2)

$$\vec{A} = \hat{i} + \hat{j} \Rightarrow |\vec{A}| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$\cos \alpha = \frac{A_x}{|\vec{A}|} = \frac{1}{\sqrt{2}} = \cos 45^\circ \therefore \alpha = 45^\circ$$

43. (3)

Fact

44. (3)

But if the angle between $\vec{A}$ and $\vec{B}$ is 0 or π . Then

$$\vec{A} \times \vec{B} = \vec{B} \times \vec{A} = \vec{0}.$$

45. (3)

$$\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma$$

$$= 1 - \cos^2 \alpha + 1 - \cos^2 \beta + 1 - \cos^2 \gamma$$

$$= 3 - (\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma) = 3 - 1 = 2$$

46. (4)

47. (2)

48. (3)

$$\sin A = \frac{1}{2}$$

$$A = 30^\circ$$

and

$$\cos B = \frac{1}{\sqrt{2}}$$

$$B = 45^\circ$$

then

$$A + B = 30 + 45 = 75^\circ$$

49. (3)

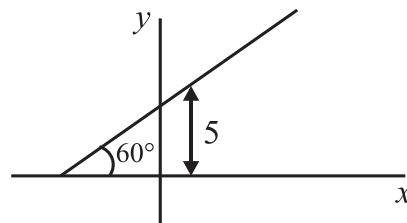
$$y = \sqrt{3}x + 5$$

$$y = mx + C$$

$$m = \tan \theta = \sqrt{3}$$

$$\boxed{\theta = 60^\circ}$$

and $C = 5$



50. (3)