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Electromagnetic Waves

Multiple Choice Questions (MCQs)

Q. 1 One requires 11 eV of energy to dissociate a carbon monoxide molecule into carbon and oxygen atoms. The minimum frequency of the appropriate electromagnetic radiation to achieve the dissociation lies in

(a) visible region
(b) infrared region
(c) ultraviolet region
(d) microwave region

Ans. *(c)* Given, energy required to dissociate a carbon monoxide molecule into carbon and oxygen atoms $E = 11\,\text{eV}$

We know that, $E = h\nu$, where $h = 6.62 \times 10^{-34}$ J-s

$$\nu = \text{frequency}$$

$$\Rightarrow \quad 11\text{eV} = h\nu$$

$$\Rightarrow \quad \nu = \frac{11 \times 1.6 \times 10^{-19}}{h}\ \text{J}$$

$$= \frac{11 \times 1.6 \times 10^{-19}}{6.62 \times 10^{-34}}\ \text{J}$$

$$= 2.65 \times 10^{15}\ \text{Hz}$$

This frequency radiation belongs to ultraviolet region.

Q. 2 A linearly polarised electromagnetic wave given as $\mathbf{E} = E_o \hat{\mathbf{i}} \cos(kz - \omega t)$ is incident normally on a perfectly reflecting infinite wall at $z = a$. Assuming that the material of the wall is optically inactive, the reflected wave will be given as

(a) $\mathbf{E}_r = E_o \hat{\mathbf{i}}(kz - \omega t)$
(b) $\mathbf{E}_r = E_o \hat{\mathbf{i}} \cos(kz + \omega t)$
(c) $\mathbf{E}_r = -E_o \hat{\mathbf{i}} \cos(kz + \omega t)$
(d) $\mathbf{E}_r = E_o \hat{\mathbf{i}} \sin(kz - \omega t)$

Thinking Process

When a wave is reflected from a denser medium, then its phase changes by 180° or π.

Ans. *(b)* When a wave is reflected from denser medium, then the type of wave doesn't change but only its phase changes by 180° or π radian.

Thus, for the reflected wave $\hat{\mathbf{z}} = -\hat{\mathbf{z}}$, $\hat{\mathbf{i}} = -\hat{\mathbf{i}}$ and additional phase of π in the incident wave.

Given, here the incident electromagnetic wave is,

$$\mathbf{E} = E_o \hat{\mathbf{i}} \cos(kz - \omega t)$$

The reflected electromagnetic wave is given by

$$\mathbf{E}_r = E_o(-\hat{\mathbf{i}})\cos[k(-z) - \omega t + \pi]$$
$$= -E_o \hat{\mathbf{i}} \cos[-(kz + \omega t) + \pi]$$
$$= E_o \hat{\mathbf{i}} \cos[-(k_Z + \omega t) = E_o \hat{\mathbf{i}} \cos(kz + \omega t)]$$

Q. 3 Light with an energy flux of 20 W/cm^2 falls on a non-reflecting surface at normal incidence. If the surface has an area of 30 cm^2, the total momentum delivered (for complete absorption) during 30 min is

(a) 36×10^{-5} kg-m/s
(b) 36×10^{-4} kg-m/s
(c) 108×10^{4} kg-m/s
(d) 1.08×10^{7} kg-m/s

Ans. *(b)* Given, energy flux $\phi = 20 \text{W} / \text{cm}^2$

Area, $A = 30 \text{cm}^2$

Time, $t = 30 \text{ min} = 30 \times 60 \text{ s}$

Now, total energy falling on the surface in time t is, $U = \phi A t = 20 \times 30 \times (30 \times 60)$ J

Momentum of the incident light $= \dfrac{U}{c}$

$$= \frac{20 \times 30 \times (30 \times 60)}{3 \times 10^8} \Rightarrow = 36 \times 10^{-4} \text{ kg-ms}^{-1}$$

Momentum of the reflected light = 0

$\therefore$ Momentum delivered to the surface

$$= 36 \times 10^{-4} - 0 = 36 \times 10^{-4} \text{ kg-ms}^{-1}$$

Q. 4 The electric field intensity produced by the radiations coming from 100 W bulb at a 3 m distance is E. The electric field intensity produced by the radiations coming from 50 W bulb at the same distance is

(a) $\dfrac{E}{2}$ (b) $2E$ (c) $\dfrac{E}{\sqrt{2}}$ (d) $\sqrt{2}E$

Thinking Process

Electric field intensity on a surface due to incident radiation is,

$$I_{av} \propto E_o^2$$
$$\frac{P_{av.}}{A} \propto E_o^2$$

Here, $P_{av} \propto E_o^2$ [$\because$ *A is same in both cases*]

Ans. *(c)* We know that, $E_o \propto \sqrt{P_{av}}$

$\therefore$ $$\frac{(E_o)_1}{(E_o)_2} = \sqrt{\frac{(P_{av})_1}{(P_{av})_2}} \Rightarrow \frac{E}{(E_o)_2} = \sqrt{\frac{1000}{50}}$$

$$(E_o)_2 = E / \sqrt{2}$$

Now according to question, $P' = 50$ W, $P = 100$ W

$\therefore$ Putting these value in Eq.(i), we get

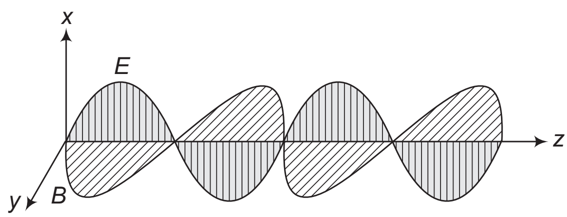
$$\frac{E'}{E} = \frac{50}{100} \Rightarrow \frac{E'}{E} = \frac{1}{2} \Rightarrow E' = \frac{E}{2}$$

Q. 5 If **E** and **B** represent electric and magnetic field vectors of the electromagnetic wave, the direction of propagation of electromagnetic wave is along

(a) **E** (b) **B** (c) **B** × **E** (d) **E** × **B**

Ans. *(d)* The direction of propagation of electromagnetic wave is perpendicular to both electric field vector **E** and magnetic field vector **B**, *i.e.,* in the direction of **E** × **B**.

This can be seen by the diagram given below



Here, electromagnetic wave is along the *z*-direction which is given by the cross product of **E** and **B**.

Q. 6 The ratio of contributions made by the electric field and magnetic field components to the intensity of an EM wave is

(a) $c:1$ (b) $c^2:1$ (c) 1 : 1 (d) $\sqrt{c}:1$

Thinking Process

Intensity of electromagnetic wave, $I = U_{av}c$

where, U_{av} = *Average energy*

and c = *speed to light*

Ans. *(c)* Intensity in terms of electric field $U_{av} = \frac{1}{2}\varepsilon_o E_o^2$

Intensity in terms of magnetic field $U_{av} = \frac{1}{2}\frac{B_o^2}{\mu_o}$

Now taking the intensity in terms of electric field.

$$(U_{av})\text{electric field} = \frac{1}{2}\varepsilon_o E_o^2$$

$$\Rightarrow \quad = \frac{1}{2}\varepsilon_o (cB_o)^2 \qquad (\because E_o = cB_o)$$

$$= \frac{1}{2}\varepsilon_o \times c^2 B^2$$

But, $$c = \frac{1}{\sqrt{\mu_o \varepsilon_o}}$$

$$\therefore \quad (U_{av})_{\text{Electric field}} = \frac{1}{2}\varepsilon_o \times \frac{1}{\mu_o \varepsilon_o} B_o^2 = \frac{1}{2}\frac{B_o^2}{\mu_o}$$

$$= (U_{av})_{\text{magnetic field}}$$

Thus, the energy in electromagnetic wave is divided equally between electric field vector and magnetic field vector.

Therefore, the ratio of contributions by the electric field and magnetic field components to the intensity of an electromagnetic wave is 1 : 1.

Q. 7 An EM wave radiates outwards from a dipole antenna, with E_o as the amplitude of its electric field vector. The electric field E_o which transports significant energy from the source falls off as

(a) $\frac{1}{r^3}$ (b) $\frac{1}{r^2}$

(c) $\frac{1}{r}$ (d) remains constant

Ans. *(c)* From a diode antenna, the electromagnetic waves are radiated outwards.

The amplitude of electric field vector (E_o) which transports significant energy from the source falls off intensity inversely as the distance (r) from the antenna, *i.e.*, $E_o \propto \frac{1}{r}$.

Q. 8 An electromagnetic wave travels in vacuum along z-direction $\mathbf{E} = (E_1\hat{\mathbf{i}}\ E_2\hat{\mathbf{j}})\cos(kz - \omega t)$. Choose the correct options from the following

(a) The associated magnetic field is given as

$$\mathbf{B} = \frac{1}{c}(E_1\hat{\mathbf{i}} - E_2\hat{\mathbf{j}})\cos(kz - \omega t)$$

(b) The associated magnetic field is given as

$$\mathbf{B} = \frac{1}{c}(E_1\hat{\mathbf{i}} - E_2\hat{\mathbf{j}})\cos(kz - \omega t)$$

(c) The given electromagnetic field is circularly polarised

(d) The given electromagnetic wave is plane polarised

K **Thinking Process**

From Maxwell's equations, it is seen that the magnitude of the electric and the magnetic fields in an electromagnetic wave are related as

$$B_o = \frac{E_o}{c}$$

Ans. *(d)* Here, in electromagnetic wave, the electric field vector is given as,

$$\mathbf{E} = (E_1\hat{\mathbf{i}} + E_2\hat{\mathbf{j}})\cos(kz - \omega t)$$

In electromagnetic wave, the associated magnetic field vector,

$$\mathbf{B} = \frac{\mathrm{E}}{c} = \frac{E_1\hat{\mathbf{i}} + E_2\hat{\mathbf{j}}}{c}\cos(kz - \omega t)$$

Also, **E** and **B** are perpendicular to each other and the propagation of electromagnetic wave is perpendicular to **E** as well as **B**, so the given electromagnetic wave is plane polarised.

Multiple Choice Questions (More Than One Options)

Q. 9 An electromagnetic wave travelling along z-axis is given as $\mathbf{E} = \mathbf{E_0} \cos(kz - \omega t)$. Choose the correct options from the following

(a) The associated magnetic field is given as $B = \frac{1}{c}\hat{\mathbf{k}} \times \mathbf{E} = \frac{1}{\omega}(\hat{\mathbf{k}} \times \mathbf{E})$

(b) The electromagnetic field can be written in terms of the associated magnetic field as $\mathbf{E} = c(\mathbf{B} \times \hat{\mathbf{k}})$

(c) $\hat{\mathbf{k}} \cdot \mathbf{E} = 0, \hat{\mathbf{k}} \cdot \mathbf{B} = 0$

(d) $\hat{\mathbf{k}} \times \mathbf{E} = 0, \hat{\mathbf{k}} \times \mathbf{B} = 0$

K **Thinking Process**

Given, $E = E_0 \cos(kz - \omega t)$. Thus, it acts along negative y-direction.

Ans. *(a, b, c)*

Suppose an electromagnetic wave is travelling along negative z-direction. Its electric field is given by

$$\mathbf{E} = E_0 \cos(kz - \omega t)$$

which is perpendicular to z-axis. It acts along negative y-direction.

The associated magnetic field **B** in electromagnetic wave is along x-axis *i.e.,* along $\hat{\mathbf{k}} \times \mathbf{E}$.

As, $$B_0 = \frac{E_0}{c}$$

$\therefore$ $$\mathbf{B} = \frac{1}{c}(\hat{\mathbf{k}} \times \mathbf{E})$$

The associated electric field can be written in terms of magnetic field as

$$\mathbf{E} = c(\mathbf{B} \times \hat{\mathbf{k}}).$$

Angle between $\hat{\mathbf{k}}$ and **E** is 90° between $\hat{\mathbf{k}}$ and **B** is 90°. Therefore, $\mathbf{E} = 1E\cos 90° = 0$ and $\hat{\mathbf{k}} \cdot \mathbf{B} = 1\,E\cos 90° = 0$.

Q. 10 A plane electromagnetic wave propagating along x-direction can have the following pairs of **E** and **B**.

(a) E_x, B_y (b) E_y, B_z (c) B_x, E_y (d) E_z, B_y

Ans. *(b, d)*

As electric and magnetic field vectors **E** and **B** are perpendicular to each other as well as perpendicular to the direction of propagation of electromagnetic wave.

Here in the question electromagnetic wave is propagating along x-direction. So, electric and magnetic field vectors should have either y-direction or z-direction.

Q. 11 A charged particle oscillates about its mean equilibrium position with a frequency of 10^9 Hz. The electromagnetic waves produced

(a) will have frequency of 10^9 Hz

(b) will have frequency of 2×10^9 Hz

(c) will have wavelength of 0.3 m

(d) fall in the region of radiowaves

K **Thinking Process**

The frequency of electromagnetic waves produced by a charged particle is equal to the frequency by which it oscillates about its mean equilibrium position.

Ans. *(a, c, d)*

Given, frequency by which the charged particles oscillates about its mean equilibrium position $= 10^9$ Hz.

So, frequency of electromagnetic waves produced by the charged particle is $\nu = 10^9$ Hz.

$$\text{Wavelength } \lambda = \frac{c}{\nu} = \frac{3 \times 10^8}{10^9} = 0.3\,\text{m}$$

Also, frequency of 10^9 Hz fall in the region of radiowaves.

Q. 12 The source of electromagnetic waves can be a charge

(a) moving with a constant velocity
(b) moving in a circular orbit
(c) at rest
(d) falling in an electric field

Thinking Process

An electromagnetic wave can be produced by accelerated or oscillating charge.

Ans. ***(b, d)***

Here, in option (b) charge is moving in a circular orbit.

In circular motion, the direction of the motion of charge is changing continuously, thus it is an accelerated motion and this option is correct.

Also, we know that a charge starts accelerating when it falls in an electric field.

Q. 13 An EM wave of intensity I falls on a surface kept in vacuum and exerts radiation pressure p on it. Which of the following are true?

(a) Radiation pressure is $\frac{I}{c}$ if the wave is totally absorbed

(b) Radiation pressure is $\frac{I}{c}$ if the wave is totally reflected

(c) Radiation pressure is $\frac{2I}{c}$ if the wave is totally reflected

(d) Radiation pressure is in the range $\frac{I}{c} < p < \frac{2I}{c}$ for real surfaces

Ans. ***(a, c, d)***

Radiation pressure (p) is the force exerted by electromagnetic wave on unit area of the surface, *i.e.,* rate of change of momentum per unit area of the surface.

Momentum per unit time per unit area

$$= \frac{\text{Intensity}}{\text{Speed of wave}} = \frac{I}{c}$$

Change in momentum per unit time per unit area $= \frac{\Delta I}{c}$ = radiation pressure (p)

i.e., $$p = \frac{\Delta I}{c}$$

Momentum of incident wave per unit time per unit area $= \frac{I}{c}$

When wave is fully absorbed by the surface, the momentum of the reflected wave per unit time per unit area = 0.

Radiation pressure (p) = change in momentum per unit time per unit area $= \frac{\Delta I}{c} = \frac{I}{c} - 0 = \frac{I}{c}$.

When wave is totally reflected, then momentum of the reflected wave per unit time per unit area $= -\frac{I}{c}$, Radiation pressure $p = \frac{I}{c} - \left(-\frac{I}{c}\right) = \frac{2I}{c}$.

Here, p lies between $\frac{I}{c}$ and $\frac{2I}{c}$.

Very Short Answer Type Questions

Q. 14 Why is the orientation of the portable radio with respect to broadcasting station important?

Ans. The orientation of the portable radio with respect to broadcasting station is important because the electromagnetic waves are plane polarised, so the receiving antenna should be parallel to the vibration of the electric or magnetic field of the wave.

Q. 15 Why does microwave oven heats up a food item containing water molecules most efficiently?

Ans. Microwave oven heats up the food items containing water molecules most efficiently because the frequency of microwaves matches the resonant frequency of water molecules.

Q. 16 The charge on a parallel plate capacitor varies as $q = q_0 \cos 2\pi vt$. The plates are very large and close together (area $= A$, separation $= d$). Neglecting the edge effects, find the displacement current through the capacitor.

Ans. The displacement current through the capacitor is,

$$I_d = I_c = \frac{dq}{dt} \quad \ldots \text{(i)}$$

Here, $q = q_0 \cos 2\pi vt$ (given)

Putting this value in Eq (i), we get

$$I_d = I_c = -q_0 \sin 2\pi vt \times 2\pi v$$
$$I_d = I_c = -2\pi v q_0 \sin 2\pi vt$$

Q. 17 A variable frequency AC source is connected to a capacitor. How will the displacement current change with decrease in frequency?

K **Thinking Process**

Capacities reactance X_c is inversely proportional to the displacement current i.e., $X_c \propto \frac{1}{I}$.

Ans. Capacitive reaction $X_c = \frac{1}{2\pi fC}$,

$$\therefore \quad X_c \propto \frac{1}{f}$$

As frequency decreases, X_c increases and the conduction current is inversely proportional to X_c $\left(\because I \propto \frac{1}{X_c}\right)$.

So, displacement current decreases as the conduction current is equal to the displacement current.

Q. 18 The magnetic field of a beam emerging from a filter facing a floodlight is given by

$$B_0 = 12 \times 10^{-8} \sin(1.20 \times 10^7 z - 3.60 \times 10^{15} t) \text{ T}.$$

What is the average intensity of the beam?

Ans. Magnetic field $\mathbf{B} = B_0 \sin\omega t$

Given, equation $B = 12 \times 10^{-8} \sin(1.20 \times 10^7 z - 3.60 \times 10^{15} t)$ T.

On comparing this equation with standard equation, we get

$$B_0 = 12 \times 10^{-8}$$

The average intensity of the beam $I_{av} = \frac{1}{2}\frac{B_0^2}{\mu_0} \cdot c = \frac{1}{2} \times \frac{(12 \times 10^{-8})^2 \times 3 \times 10^8}{4\pi \times 10^{-7}}$

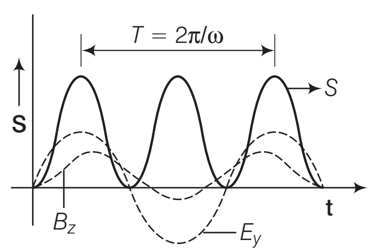
$$= 1.71 \text{ W/m}^2$$

Q. 19 Poynting vectors **S** is defined as a vector whose magnitude is equal to the wave intensity and whose direction is along the direction of wave propogation. Mathematically, it is given by $\mathbf{S} = \frac{1}{\mu_o} \mathbf{E} \times \mathbf{B}$. Show the nature of **S** *versus* t graph.

Ans. Consider and electromagnetic waves, let **E** be varying along y-axis, **B** is along z-axis and propagation of wave be along x-axis. Then $\mathbf{E} \times \mathbf{B}$ will tell the direction of propagation of energy flow in electromegnetic wave, along x-axis.

Let
$$\mathbf{E} = E_o \sin(\omega t - kx)\hat{\mathbf{j}}$$
$$\mathbf{B} = B_o \sin(\omega t - kx)\hat{\mathbf{k}}$$
$$\mathbf{S} = \frac{1}{\mu_o}(\mathbf{E} \times \mathbf{B}) = \frac{1}{\mu_o} E_o B_o \sin^2(\omega t - kx)[\hat{\mathbf{j}} \times \hat{\mathbf{k}}]$$
$$= \frac{E_o B_o}{\mu_o} \sin^2(\omega t - kx)\,\hat{\mathbf{i}}$$

The variation of $|\mathbf{S}|$ with time t will be as given in the figure below



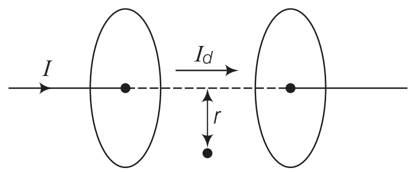
Q. 20 Professor CV Raman surprised his students by suspending freely a tiny light ball in a transparent vacuum chamber by shining a laser beam on it. Which property of EM waves was he exhibiting? Give one more example of this property.

Ans. An electromagnetic wave carries energy and momentum like other waves.

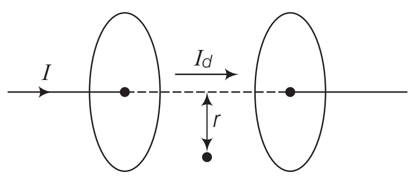
Since, it carries momentum, an electromagnetic wave also exerts pressure called radiation pressure. This property of electromagnetic waves helped professor CV Raman surprised his students by suspending freely a tiny light ball in a transparent vacuum chamber by shining a laser beam on it. The tails of the camets are also due to radiation pressure.

Short Answer Type Questions

Q. 21 Show that the magnetic field B at a point in between the plates of a parallel plate capacitor during charging is $\frac{\mu_o \varepsilon_o r}{2}\frac{dE}{dt}$ (symbols having usual meaning).



Ans. Consider the figure ginen below to prove that the magneti field B at a point in between the plater of a paravel- plate copocior during charging is $\frac{\varepsilon_0 \mu_0 r}{2}\frac{dE}{dt}$



Let I_d be the displacement current in the region between two plates of parallel plate capacitor, in the figure.

The magnetic field induction at a point in a region between two plates of capacitor at a perpendicular distance r from the axis of plates is

$$B = \frac{\mu_0 2 I_d}{4\pi r} = \frac{\mu_0}{2\pi r} I_d = \frac{\mu_0}{2\pi r} \times \varepsilon_0 \frac{d\phi_E}{dt} \qquad \left[\because I_d = \frac{E_0 d\,\phi_E}{dt}\right]$$

$$= \frac{\mu_0 \varepsilon_0}{2\pi r}\frac{d}{dt}(E\pi r^2) = \frac{\mu_0 \varepsilon_0}{2\pi r}\pi r^2 \frac{dE}{dt}$$

$$B = \frac{\mu_0 \varepsilon_0 r}{2}\frac{dE}{dt} \qquad [\because \phi_E = E\pi r^2]$$

Q. 22 Electromagnetic waves with wavelength

(i) λ_1 is used in satellite communication.

(ii) λ_2 is used to kill germs in water purifies.

(iii) λ_3 is used to detect leakage of oil in underground pipelines.

(iv) λ_4 is used to improve visibility in runways during fog and mist conditions.

(a) Identify and name the part of electromagnetic spectrum to which these radiations belong.

(b) Arrange these wavelengths in ascending order of their magnitude.

(c) Write one more application of each.

Ans. (a) (i) Microwave is used in satellite communications.

So, λ_1 is the wavelength of microwave.

(ii) Ultraviolet rays are used to kill germs in water purifier. So, λ_2 is the wavelength of UV rays.

(iii) X-rays are used to detect leakage of oil in underground pipelines. So, λ_3 is the wavelength of X-rays.

(iv) Infrared is used to improve visibility on runways during fog and mist conditions. So, it is the wavelength of infrared waves.

(b) Wavelength of X-rays < wavelength of UV < wavelength of infrared < wavelength of microwave.

$$\Rightarrow \qquad \lambda_3 < \lambda_2 < \lambda_4 < \lambda_1$$

(c) (i) Microwave is used in radar.

(ii) UV is used in LASIK eye surgery.

(iii) X-ray is used to detect a fracture in bones.

(iv) Infrared is used in optical communication.

Q. 23 Show that average value of radiant flux density S over a single period T is given by $S = \dfrac{1}{2c\mu_0} E_0^2$.

Ans. Radiant flux density $S = \dfrac{1}{\mu_0}(\mathbf{E} \times \mathbf{B}) = c^2 \varepsilon_o (\mathbf{E} \times \mathbf{B})$ $\qquad \left[\because c = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}}\right]$

Suppose electromagnetic waves be propagating along x-axis. The electric field vector of electromagnetic wave be along y-axis and magnetic field vector be along z-axis. Therefore,

$$\mathbf{E}_0 = \mathbf{E}_0 \cos(kx - \omega t)$$

and

$$\mathbf{B} = \mathbf{B}_0 \cos(kx - \omega t)$$

$$\mathbf{E} \times \mathbf{B} = (\mathbf{E}_0 \times \mathbf{B}_0)\cos^2(kx - \omega t)$$

$$\mathbf{S} = \mathbf{c^2} \varepsilon_o (\mathbf{E} \times \mathbf{B})$$

$$= c^2 \varepsilon_0 (\mathbf{E_o} \times \mathbf{B_o})\cos^2(kx - \omega t)$$

Average value of the magnitude of radiant flux density over complete cycle is

$$S_{av} = c^2 \varepsilon_o |\mathbf{E}_0 \times \mathbf{B}_0| \frac{1}{T}\int_o^T \cos^2(kx - \omega t)dt$$

$$= c^2 \varepsilon_0 E_0 B_0 \times \frac{1}{T} \times \frac{T}{2} \qquad \left[\because \int_o^T \cos^2(kx - \omega t)dt = \frac{T}{2}\right]$$

$$\Rightarrow \qquad S_{av} = \frac{c^2}{2} \varepsilon_0 E_0 \left(\frac{E_0}{c}\right) \qquad \left[\text{As, } c = \frac{E_0}{B_0}\right]$$

$$= \frac{c}{2} \varepsilon_0 E_0^2 = \frac{c}{2} \times \frac{1}{c^2 \mu_0} E_0^2 \qquad \left[c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \text{ or } \varepsilon_0 = \frac{1}{c^2 \mu_0}\right]$$

$$\Rightarrow \qquad S_{av} = \frac{E_0^2}{2\mu_0 c}$$

Hence proved.

Q. 24 You are given a 2μF parallel plate capacitor. How would you establish an instantaneous displacement current of 1 mA in the space between its plates?

Ans. Given, capacitance of capacitor $C = 2\mu F$,

Displacement current $I_d = 1\,mA$

Charge $q = CV$

$$I_d dt = CdV \qquad [\because q = it]$$

or $$I_d = C\frac{dV}{dt}$$

$$1 \times 10^{-3} = 2 \times 10^{-6} \times \frac{dV}{dt}$$

or $$\frac{dV}{dt} = \frac{1}{2} \times 10^{+3} = 500\,V/s$$

So, by applying a varying potential difference of 500 V/s, we would produce a displacement current of desired value.

Q. 25 Show that the radiation pressure exerted by an EM wave of intensity I on a surface kept in vacuum is $\frac{I}{C}$.

Ans. Pressure $= \frac{\text{Force}}{\text{Area}} = \frac{F}{A}$

Force is the rate of change of momentum

i.e., $$F = \frac{dp}{dt}$$

Energy in time dt, $$U = p \cdot C \text{ or } p = \frac{U}{c}$$

$\therefore$ $$\text{Pressure} = \frac{1}{A} \cdot \frac{U}{C \cdot dt}$$

$$\text{Pressure} = \frac{I}{C} \qquad \left[\because I = \text{Intensity} = \frac{U}{A \cdot dt}\right]$$

Q. 26 What happens to the intensity of light from a bulb if the distance from the bulb is doubled? As a laser beam travels across the length of room, its intensity essentially remain constant.

What geometrical characteristic of LASER beam is responsible for the constant intensity which is missing in the case of light from the bulb?

Ans. As the distance is doubled, the area of spherical region $(4\pi r^2)$ will become four times, so the intensity becomes one fourth the initial value $\left(\because I \propto \frac{1}{r^2}\right)$ but in case of laser it does not spread, so its intensity remain same.

Geometrical characteristic of LASER beam which is responsible for the constant intensity are as following

(i) Unidirection (ii) Monochromatic

(iii) Coherent light (iv) Highly collimated

These characteristic are missing in the case of light from the bulb.

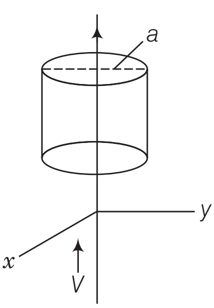
Q. 27 Even though an electric field E exer ts a force qE on a charged particle yet electric field of an EM wave does not contribute to the radiation pressure (but transfers energy). Explain.

Ans. Since, electric field of an EM wave is an oscillating field and so is the electric force caused by it on a charged particle. This electric force averaged over an integral number of cycles is zero, since its direction changes every half cycle.

Hence, electric field is not responsible for radiation pressure.

Long Answer Type Questions

Q. 28 An infinitely long thin wire carrying a uniform linear static charge density λ is placed along the z-axis (figure). The wire is set into motion along its length with a uniform velocity $v = v\hat{\mathbf{k}}_z$. Calculate the pointing vector $\mathbf{S} = \frac{1}{\mu_0}(\mathbf{E} \times \mathbf{B})$.



Ans. Given,

$$\mathbf{E} = \frac{\lambda \hat{\mathbf{e}}_s}{2\pi\varepsilon_0 a}\hat{\mathbf{j}}$$

$$\mathbf{B} = \frac{\mu_0 i}{2\pi a}\hat{\mathbf{i}} = \frac{\mu_0 \lambda v}{2\pi a}\hat{\mathbf{i}} \qquad [\because I = \lambda_V]$$

$$\therefore \quad \mathbf{S} = \frac{1}{\mu_0}[\mathbf{E} \times \mathbf{B}] = \frac{1}{\mu_0}\left[\frac{\lambda_{\hat{j}}}{2\pi\varepsilon_0 a} \times \frac{\mu_0}{2\pi a}\lambda\sqrt{\hat{\mathbf{i}}}\right]$$

$$= \frac{\lambda^2 v}{4\pi^2\varepsilon_0 a^2}(\hat{\mathbf{j}} \times \hat{\mathbf{i}}) = -\frac{\lambda^2 V}{4\pi^2\varepsilon_0 a^2}\hat{\mathbf{k}}$$

Q. 29 Sea water at frequency $v = 4 \times 10^8$ Hz has permittivity $\varepsilon \approx 80\varepsilon_0$, permeability $\mu \approx \mu_0$ and resistivity $\rho = 0.25$ m. Imagine a parallel plate capacitor immersed in sea water and driven by an alternating voltage source $V(t) = V_0 \sin(2\pi vt)$. What fraction of the conduction current density is the displacement current density?

K **Thinking Process**

The conduction current density is given by the Ohm's law = Electric field between the plates.

Ans. Suppose distance between the parallel plates is d and applied voltage $V_{(t)} = V_0 2\pi vt$.
Thus, electric field

$$E = \frac{V_0}{d}\sin(2\pi vt)$$

Now using Ohm's law, $$J_c = \frac{1}{\rho}\frac{V_0}{d}\sin(2\pi vt)$$

$\Rightarrow$ $$= \frac{V_0}{\rho d}\sin(2\pi vt) = J_0^c \sin 2\pi vt$$

Here, $$J_0^c = \frac{V_0}{\rho d}$$

Now the displacement current density is given as

$$J_d = \varepsilon\frac{\delta E}{dt} = \frac{\varepsilon\delta}{dt}\left[\frac{V_0}{d}\sin(2\pi vt)\right]$$

$$= \frac{\varepsilon 2\pi v V_0}{d}\cos(2\pi vt)$$

$\Rightarrow$ $$= J_0^d \cos(2\pi vt)$$

where, $$J_0^d = \frac{2\pi V\varepsilon V_0}{d}$$

$\Rightarrow$ $$\frac{J_0^d}{J_0^c} = \frac{2\pi v\varepsilon V_0}{d}\cdot\frac{\rho d}{V_0} = 2\pi v\varepsilon\rho$$

$$= 2\pi \times 80\varepsilon_0 v \times 0.25 = 4\pi\varepsilon_0 v \times 10$$

$$= \frac{10v}{9\times 10^9} = \frac{4}{9}$$

Q. 30 A long straight cable of length l is placed symmetrically along z-axis and has radius $a(<< l)$. The cable consists of a thin wire and a co-axial conducting tube. An alternating current $I(t) = I_0 \sin(2\pi vt)$ flows down the central thin wire and returns along the co-axial conducting tube. The induced electric field at a distance s from the wire inside the cable is

$$\mathbf{E}(s,t) = \mu_0 I_0 v\cos(2\pi vt)\ln\left(\frac{s}{a}\right)\hat{\mathbf{k}}.$$

(i) Calculate the displacement current density inside the cable.

(ii) Integrate the displacement current density across the cross-section of the cable to find the total displacement current I^d.

(iii) Compare the conduction current I_0 with the displacement current I_0^d.

K **Thinking Process**

Displacement current density

$$\mathbf{J}_d = \varepsilon_0\frac{d\mathbf{E}}{dt}$$

Ans. **(i)** Given, the induced electric field at a distance r from the wire inside the cable is

$$\mathbf{E}(s,t) = \mu_0 I_0 v\cos(2\pi vt)\ln\left(\frac{s}{a}\right)\hat{\mathbf{k}}$$

Now, displacement current density,

$$\mathbf{J}_d = \varepsilon_0\frac{d\mathbf{E}}{dt} = \varepsilon_0\frac{d}{dt}\left[\mu_0 I_0 v\cos(2\pi vt)\ln\left(\frac{s}{a}\right)\hat{\mathbf{k}}\right]$$

$$= \varepsilon_0 \mu_0 I_0 v \frac{d}{dt}[\cos 2\pi vt] \ln\left(\frac{s}{a}\right)\hat{\mathbf{k}}$$

$$= \frac{1}{c^2} I_0 v^2 2\pi[-\sin 2\pi vt] \ln\left(\frac{s}{a}\right)\hat{\mathbf{k}}$$

$$= \frac{v^2}{c^2} 2\pi I_0 \sin 2\pi vt \ \ln\left(\frac{a}{s}\right)\hat{\mathbf{k}} \qquad \left[\because l_4 \frac{s}{a} = -l_4 \frac{a}{s}\right]$$

$$= \frac{1}{\lambda^2} 2\pi I_0 \ln\left(\frac{a}{s}\right) \sin 2\pi vt \ \hat{\mathbf{k}}$$

$$= \frac{2\pi I_0}{\lambda_2} \ln\frac{a}{s} \sin 2\pi vt \ \hat{\mathbf{k}}$$

(ii)

$$I_d = \int J_d \, s ds d\theta = \int_{s=0}^{a} J_d \, s ds \int_0^{2\pi} d\theta = \int_{s=0}^{a} J_d \, s ds \times 2\pi$$

$$= \int_{s=0}^{a} \left[\frac{2\pi}{\lambda^2} I_0 \log_e\left(\frac{a}{s}\right) s ds \sin 2\pi vt\right] \times 2\pi$$

$$= \left(\frac{2\pi}{\lambda}\right)^2 I_0 \int_{s=0}^{a} \left(\frac{a}{s}\right) s ds \sin 2\pi vt$$

$$\Rightarrow \quad = \left(\frac{2\pi}{\lambda}\right)^2 I_0 \int_{s=0}^{a} \ln\left(\frac{a}{s}\right) \frac{1}{2} d(s^2) . \sin 2\pi vt$$

$$= \frac{a^2}{2}\left(\frac{2\pi}{\lambda}\right)^2 I_0 \sin 2\pi vt \int_{s=0}^{a} \ln\left(\frac{a}{s}\right) \cdot d\left(\frac{s}{a}\right)^2$$

$$= \frac{a^2}{4}\left(\frac{2\pi}{\lambda}\right)^2 I_0 \sin 2\pi vt \int_{s=0}^{a} \ln\left(\frac{a}{s}\right)^2 \cdot d\left(\frac{s}{a}\right)^2$$

$$= -\frac{a^2}{4}\left(\frac{2\pi}{\lambda}\right)^2 I_0 \sin 2\pi vt \int_{s=0}^{a} \ln\left(\frac{s}{a}\right)^2 \cdot d\left(\frac{s}{a}\right)^2$$

$$= -\frac{a^2}{4}\left(\frac{2\pi}{\lambda}\right)^2 I_0 \sin 2\pi vt \times (-1) \qquad \left[\because \int_{s=0}^{a} \ln\left(\frac{s}{a}\right)^2 d\left(\frac{s}{a}\right)^2 = -1\right]$$

$$\therefore \quad I_d = \frac{a^2}{4}\left(\frac{2\pi}{\lambda}\right)^2 I_0 \sin 2\pi vt$$

$$\Rightarrow \quad = \left(\frac{2\pi a}{2\lambda}\right)^2 I_0 \sin 2\pi vt$$

(iii) The displacement current,

$$I_d = \left(\frac{2\pi a}{2\lambda}\right)^2 I_0 \sin 2\pi vt = I_{0d} \sin 2\pi vt$$

Here,

$$I_{0d} = \left(\frac{2\pi a}{2\lambda}\right)^2 I_0 = \left(\frac{a\pi}{\lambda}\right)^2 I_0$$

$$\therefore \quad \frac{I_{0d}}{I_0} = \left(\frac{a\pi}{\lambda}\right)^2$$

Q. 31 A plane EM wave travelling in vacuum along z-direction is given by $\mathbf{E} = E_0 \sin(kz - \omega t)\hat{\mathbf{i}}$ and $\mathbf{B} = B_0 \sin(kz - \omega t)\hat{\mathbf{j}}$.

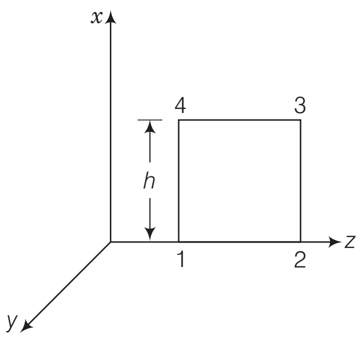
(i) Evaluate $\int \mathbf{E} \cdot \mathbf{dl}$ over the rectangular loop 1234 shown in figure.

(ii) Evaluate $\int \mathbf{B} \cdot \mathbf{ds}$ over the surface bounded by loop 1234.

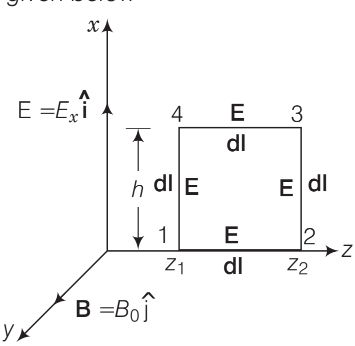
(iii) Use equation $\int \mathbf{E} \cdot \mathbf{dl} = \dfrac{-d\phi_B}{dt}$ to prove $\dfrac{E_0}{B_0} = c$.

(iv) By using similar process and the equation

$\int \mathbf{B} \cdot \mathbf{dl} = \mu_0 I + \varepsilon_0 \dfrac{d\phi_E}{dt}$, prove that $c = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}}$



Ans. **(i)** *Consider the figure given below*



During the propagation of electromagnetic wave a long z-axis, let electric field vector (**E**) be along x-axis and magnetic field vector **B** along y-axis, *i.e.*, $\mathbf{E} = E_0\hat{\mathbf{i}}$ and $\mathbf{B} = B_0\hat{\mathbf{j}}$.

Line integral of E over the closed rectangular path 1234 in x-z plane of the figure

$$\oint \mathbf{E} \cdot \mathbf{dl} = \int_1^2 \mathbf{E} \cdot \mathbf{dl} + \int_2^3 \mathbf{E} \cdot \mathbf{dl} + \int_3^4 \mathbf{E} \cdot \mathbf{dl} + \int_4^1 \mathbf{E} \cdot \mathbf{dl}$$

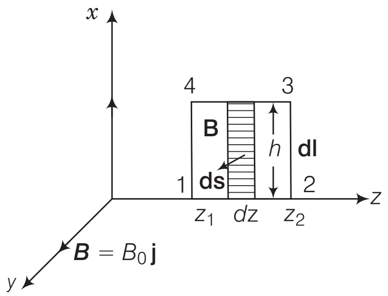
$$= \int_1^2 \mathbf{E} \cdot \mathbf{dl} \cos 90 + \int_2^3 \mathbf{E} \cdot \mathbf{dl} \cos 0 + \int_3^4 \mathbf{E} \cdot \mathbf{dl} \cos 90 + \int_4^1 \mathbf{E} \cdot \mathbf{dl} \cos 180°$$

$$= E_0 h[\sin(kz_2 - \omega t) - \sin(kz_1 - g\omega t)]$$

(ii) For evaluating $\int \mathbf{B} \cdot \mathbf{ds}$, let us consider the rectangle 1234 to be made of strips of are $ds = hdz$ each.

$$\int \mathbf{B} \cdot \mathbf{ds} = \int \mathbf{B} \cdot \mathbf{ds} \cos 0 = \int \mathbf{B} \cdot \mathbf{ds} = \int_{Z_1}^{Z_2} B_0 \sin(kz - \omega t) h dz$$

$$= \frac{-B_0 h}{k} [\cos(kz_2 - \omega t) - \cos(kz_1 - \omega t)]$$



(iii) Given, $\oint \mathbf{E} \cdot \mathbf{dl} = \frac{-d\phi_B}{dt} = -\frac{d}{dt} \oint \mathbf{B} \cdot \mathbf{ds}$

Putting the values from Eqs. (i) and (ii), we get

$$E_0 h[\sin(kz_2 - \omega t) - \sin(kz_1 - \omega t)]$$

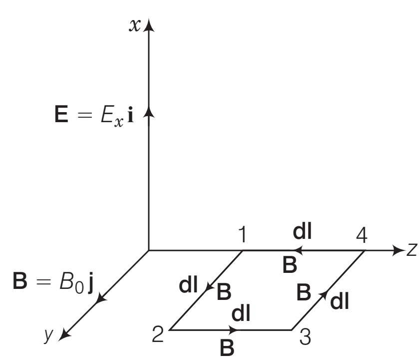
$$= \frac{-d}{dt}\left[\frac{B_y h}{k}\{\cos(kz_2 - \omega t) - \cos(kz_1 - \omega t)\}\right]$$

$$= \frac{B_y h}{k} \omega [\sin(kz_2 - \omega t) - \sin(kz_1 - \omega t)]$$

$\Rightarrow$ $$E_0 = \frac{B_0 \omega}{k} = B_y c \qquad \left(\because \frac{\omega}{k} = c\right)$$

$\Rightarrow$ $$\frac{E_0}{B_0} = c$$

(iv) For evaluating $\oint \mathbf{B} \cdot \mathbf{dl}$, let us consider a loop 1234 in y-z plane as shown in figure given below

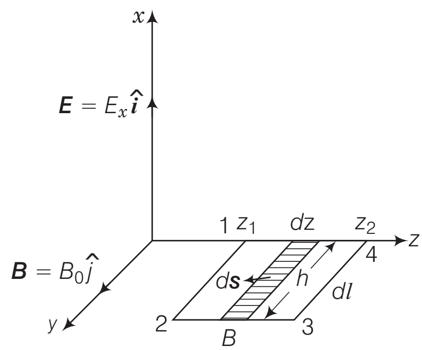


$$\oint \mathbf{B} \cdot \mathbf{dl} = \int_1^2 \mathbf{B} \cdot \mathbf{dl} + \int_2^3 \mathbf{B} \cdot \mathbf{dl} + \int_3^4 \mathbf{B} \cdot \mathbf{dl} + \int_4^1 \mathbf{B} \cdot \mathbf{dl}$$

$$= \int_1^2 \mathbf{B} \cdot \mathbf{dl} \cos 0 + \int_2^3 \mathbf{B} \cdot \mathbf{dl} \cos 90^\circ + \int_3^4 \mathbf{B} \cdot \mathbf{dl} \cos 180^\circ + \int_4^1 \mathbf{B} \cdot \mathbf{dl} \cos 90^\circ$$

$$= B_0 h[\sin(kz_1 - \omega t) - \sin(kz_2 - \omega t)] \qquad \ldots\text{(iii)}$$

Now to evaluate $\phi_E = \int \mathbf{E} \cdot \mathbf{ds}$, let us consider the rectangle 1234 to be made of strips of area hd_2 each.



$$\phi_E = \int \mathbf{E} \cdot \mathbf{ds} = \int E ds \cos 0 = \int E ds = \int_{z_1}^{z_2} E_0 \sin(kz_1 - \omega t) h dz$$

$$= -\frac{E_0 h}{k}[\cos(kz_2 - \omega t) - \cos(kz_1 - \omega t)]$$

$\therefore$
$$\frac{d\phi_E}{dt} = \frac{E_0 h\omega}{k}[\sin(kz_1 - \omega t) - \sin(kz_2 - \omega t)] \quad \ldots\text{(iv)}$$

In
$$\oint \mathbf{B} \cdot \mathbf{dl} = \mu_0 \left(I + \frac{\varepsilon_0 d\phi_E}{dt} \right), \; I = \text{conduction current}$$

$$= 0 \text{ in vacuum}$$

$\therefore$
$$\oint \mathbf{B} \cdot \mathbf{dl} = \mu_0 \varepsilon \frac{d\,\phi_E}{dt}$$

Using relations obtained in Eqs. (iii) and (iv) and simplifying, we get

$$B_0 = E_0 \frac{\omega \mu_0 \varepsilon_0}{k}$$

$\Rightarrow$
$$\frac{E_0}{B_0} \frac{\omega}{k} = \frac{1}{\mu_0 \varepsilon_0}$$

But
$$\frac{E_0}{B_0} = c \text{ and } \omega = ck$$

$\Rightarrow$
$$c \cdot c = \frac{1}{\mu_0 \varepsilon_0}, \text{ therefore } c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

Q. 32 A plane EM wave travelling along z-direction is described by $\mathbf{E} = E_0 \sin(kz - \omega t)\hat{\mathbf{i}}$ and $\mathbf{B} = B_0 \sin(kz - \omega t)\hat{\mathbf{j}}$. Show that

(i) the average energy density of the wave is given by
$$u_{av} = \frac{1}{4}\varepsilon_0 E_0^2 + \frac{1}{4}\frac{B_0^2}{\mu_0}$$

(ii) the time averaged intensity of the wave is given by
$$I_{av} = \frac{1}{2} c\varepsilon_o E_0^2.$$

Ans. **(i)** The electromagnetic wave carry energy which is due to electric field vector and magnetic field vector. In electromagnetic wave, E and B vary from point to point and from moment to moment. Let E and B be their time averages.

The energy density due to electric field E is

$$u_E = \frac{1}{2}\varepsilon_0 E^2$$

The energy density due to magnetic field B is

$$u_B = \frac{1}{2}\frac{B^2}{\mu_0}$$

Total average energy density of electromagnetic wave

$$u_{av} = u_E + u_B = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\frac{B^2}{\mu_0}$$

Let the EM wave be propagating along z-direction. The electric field vector and magnetic field vector be represented by

$$E = E_0 \sin(kz - \omega t)$$
$$B = B_0 \sin(kz - \omega t)$$

The time average value of E^2 over complete cycle $= \frac{E_0^2}{2}$

and time average value of B^2 over complete cycle $= \frac{B_0^2}{2}$

$$u_{av} = \frac{1}{2}\frac{\varepsilon_0 E_0^2}{2} + \frac{1}{2}\mu_0\left(\frac{B_0^2}{2}\right)$$

$$= \frac{1}{4}^E \varepsilon_0 E_0^2 + \frac{B_0^2}{4\mu_0}$$

(ii) We know that $E_0 = cB_0$ and $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

$\therefore$ $$\frac{1}{4}\frac{B_0^2}{\mu_0} = \frac{1}{4}\frac{E_0^2/c^2}{\mu_0} = \frac{E_0^2}{4\mu_0} \times \mu_0\varepsilon_0 = \frac{1}{4}\varepsilon_0 E_0^2$$

$\therefore$ $$u_B = u_E$$

Hence, $$u_{av} = \frac{1}{4}\varepsilon_0 E_0^2 + \frac{1}{4}\frac{B_0^2}{\mu_0}$$

$$= \frac{1}{4}\varepsilon_0 E_0^2 + \frac{1}{4}\varepsilon_0 E_0^2$$

$$= \frac{1}{2}\varepsilon_0 E_0^2 = \frac{1}{2}\frac{B_0^2}{\mu_0}$$

Time average intensity of the wave

$$I_{av} = u_{av} c = \frac{1}{2}\varepsilon_0 E_0^2 c = \frac{1}{2}\varepsilon_0 E_0^2$$