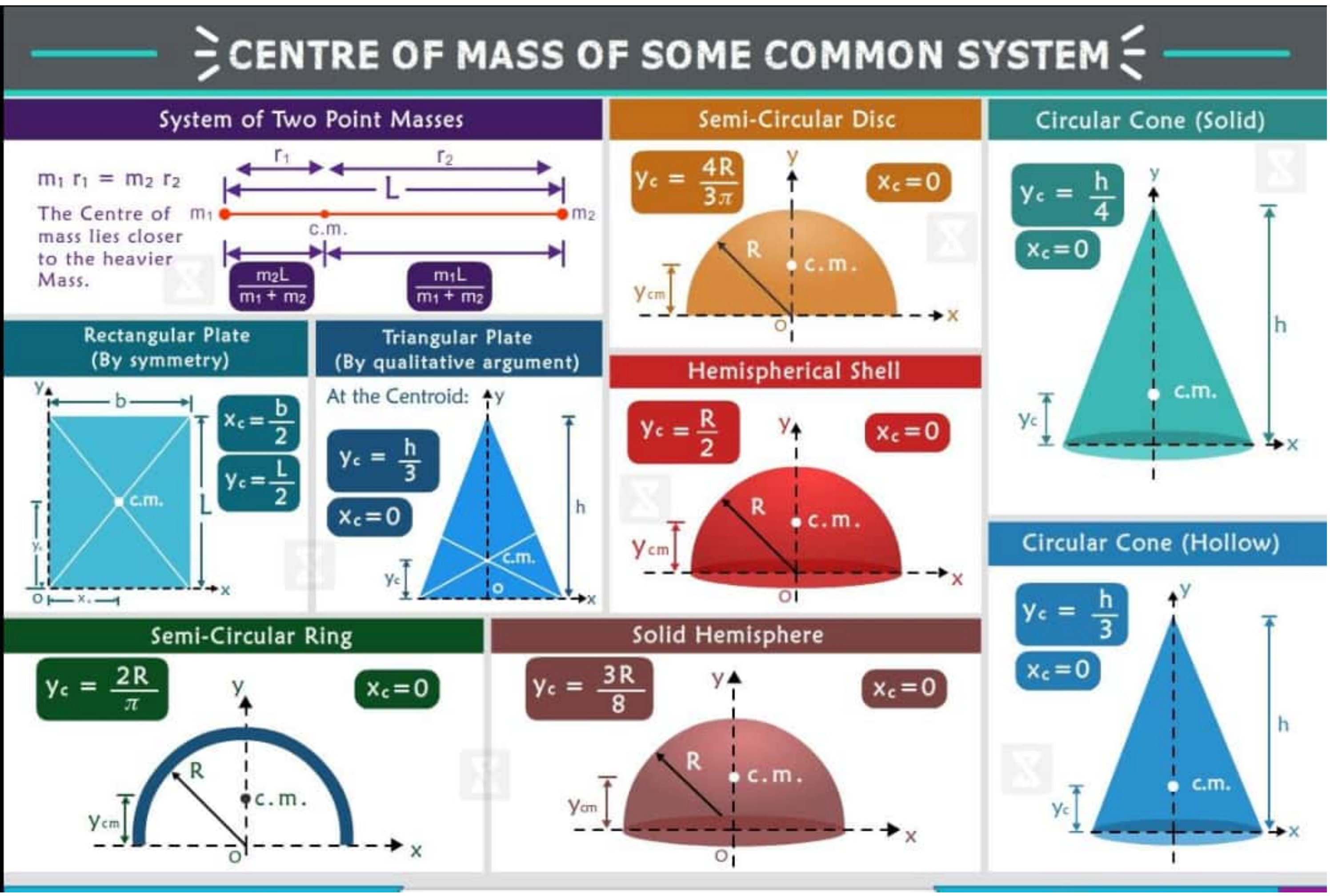
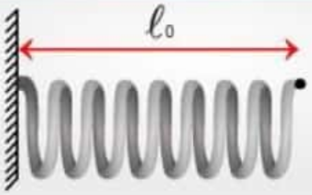

CENTRE OF MASS OF SOME COMMON SYSTEM
System of Two Point Masses
m₁ r₁ = m₂ r₂
The Centre of mass lies closer to the heavier Mass.
r₁
r₂
L
m₁
m₂
c.m.
m₂L / (m₁ + m₂)
m₁L / (m₁ + m₂)
Semi-Circular Disc
yc = 4R/3π
xc=0
R
c.m.
ycm
Circular Cone (Solid)
yc = h/4
xc=0
h
c.m.
yc
Rectangular Plate (By symmetry)
b
L
c.m.
xc = b/2
yc = L/2
Triangular Plate (By qualitative argument)
At the Centroid:
yc = h/3
xc=0
c.m.
h
yc
Hemispherical Shell
yc = R/2
xc=0
R
c.m.
ycm
Circular Cone (Hollow)
yc = h/3
xc=0
h
c.m.
yc
Semi-Circular Ring
yc = 2R/π
xc=0
R
c.m.
ycm
Solid Hemisphere
yc = 3R/8
xc=0
R
c.m.
ycm

SPRING FORCE

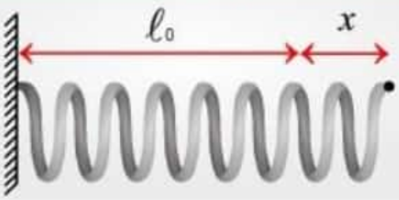
1 STRETCHED SPRING

Initial length (ℓ_0)



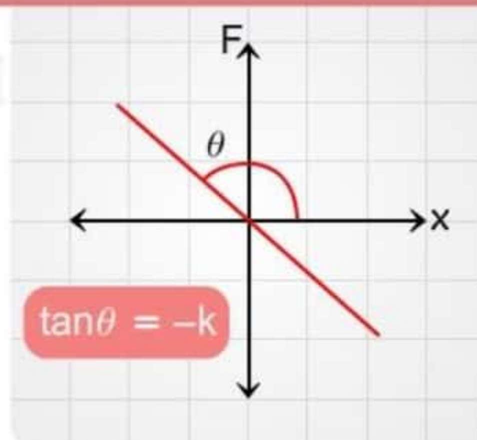
$F = 0$

Stretched by x



$F = -kx$

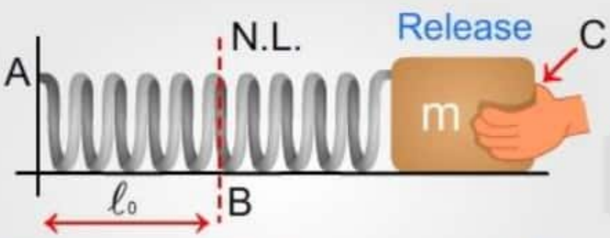
Spring Force v/s Displacement



$\tan\theta = -k$

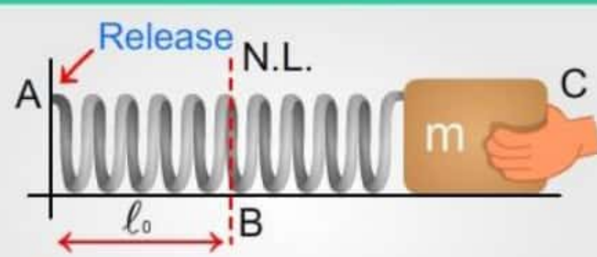
2 SPRING ATTACHED TO A BLOCK

Released at C



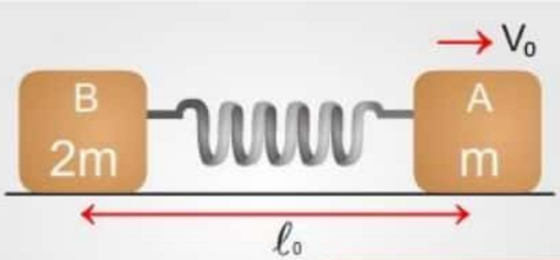
When the block is released at point C then spring force doesn't change instantaneously because of friction at mass m.

Released at A

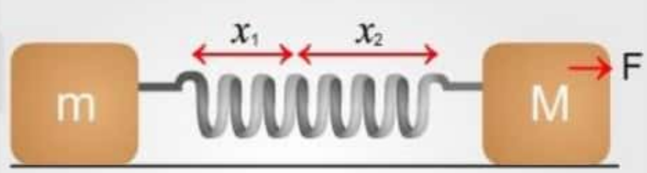


When point A is released then the spring force changes instantaneously to become zero.

3 SPRING BLOCK SYSTEM



Maximum Extension $x_{max} = V_0\sqrt{\frac{2}{3k}m}$



$$x_{max} = x_1 + x_2 = \frac{2mF}{k(m+M)}$$

IMPULSE AND MOMENTUM

IMPULSE

Impulse of a force 'F' acting on a body for a time interval $t = t_1$ to $t = t_2$ is defined as

$$\vec{I} = \int_{t_1}^{t_2} \vec{F}\, dt \qquad \vec{I}_{Re} = \int_{t_1}^{t_2} \vec{F}_{Res}\, dt = \Delta \vec{P}$$

(Impulse - Momentum Theorem)

COEFFICIENT OF RESTITUTION (e)

The coefficient of restitution is defined as the ratio of the impulses of reformation and deformation of either body.

$$e = \frac{\text{Impulse of reformation}}{\text{Impulse of deformation}} = \frac{\int F_r\, dt}{\int F_d\, dt} \qquad e = \frac{\text{Velocity of separation of point of contact}}{\text{Velocity of approach of point of contact}}$$

LINEAR MOMENTUM

Linear momentum is a vector quantity defined as the product of an object's mass m, and its velocity v. Linear momentum is denoted by the letter p and is called "momentum" in short:

$$p = mv$$

Note that a body's momentum is always in the same direction as its velocity vector. The units of momentum are kg.m/s.

CONSERVATION OF LINEAR MOMENTUM

For a single mass or single body, If net force acting on the body is zero. Then,

$$\vec{p} = \text{constant} \quad \text{or} \quad \vec{v} = \text{constant}$$

(if mass = constant)

If net external force acting on a system of particles or system of rigid bodies is zero. Then,

$$\vec{P}_{CM} = \text{constant} \quad \text{or} \quad \vec{V}_{CM} = \text{constant}$$

COLLISION

Note :- In every type of collision, only linear momentum remains constant.

HEAD ON ELASTIC COLLISION

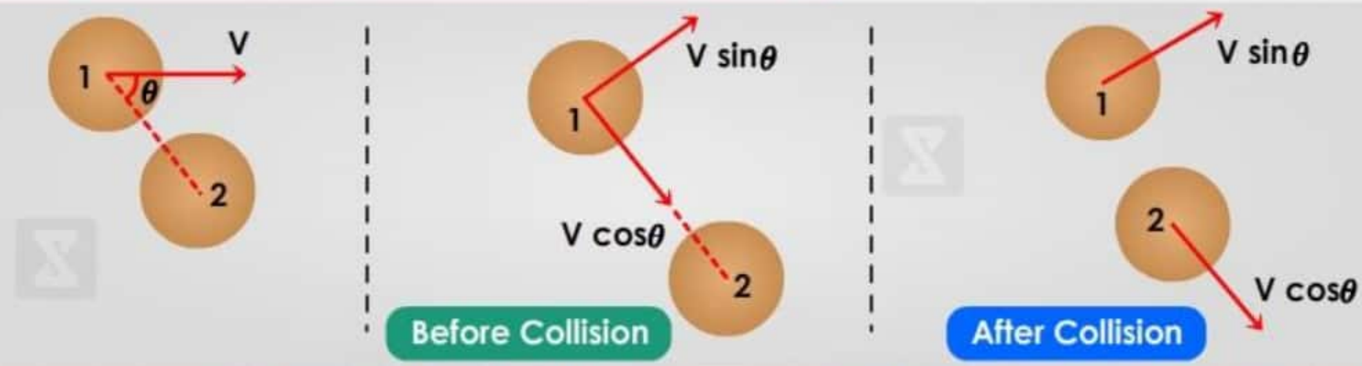
In this case, linear momentum and kinetic energy both are conserved. After solving two conservation equations. We get,

$$V'_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)V_1 + \left(\frac{2m_2}{m_1 + m_2}\right)V_2$$ and $$V'_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)V_2 + \left(\frac{2m_2}{m_1 + m_2}\right)V_1$$

HEAD ON INELASTIC COLLISION

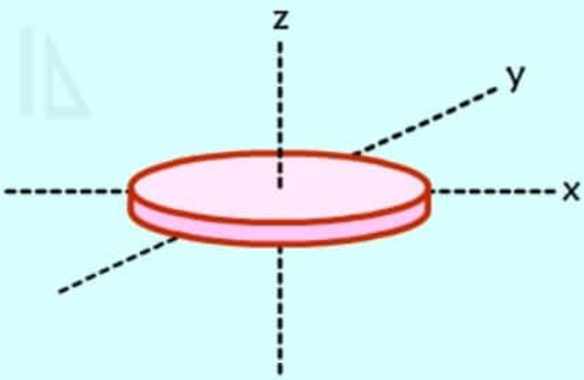
- In an inelastic collision, the colliding particles do not regain their shape and size completely after the collision.
- Some fraction of mechanical energy is retained by the colliding particles in the form of deformation potential energy. Thus, the kinetic energy of the particles no longer remains conserved.
- $(\text{Energy loss})_{\text{Perfectly Inelastic}} > (\text{Energy loss})_{\text{Partial Inelastic}}$
- $0 < e < 1$: e = coefficient of restitution

OBLIQUE COLLISION (BOTH ELASTIC IN ELASTIC)



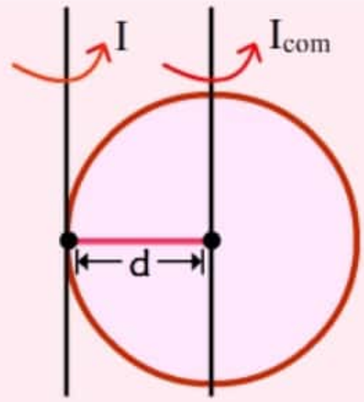
BALL	COMPONENT ALONG COMMON TANGENT DIRECTION		COMPONENT ALONG COMMON NORMAL DIRECTION	
	Before Collision	After Collision	Before Collision	After Collision
1	$V \sin\theta$	$V \sin\theta$	$V \cos\theta$	0
2	0	0	0	$V \cos\theta$

Perpendicular Axis Theorm



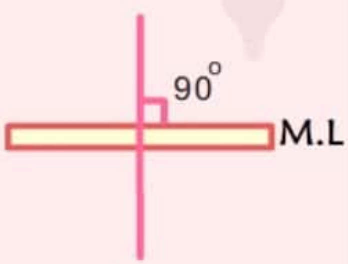
$$I_z = I_x + I_y$$

Parellel Axis Theorm



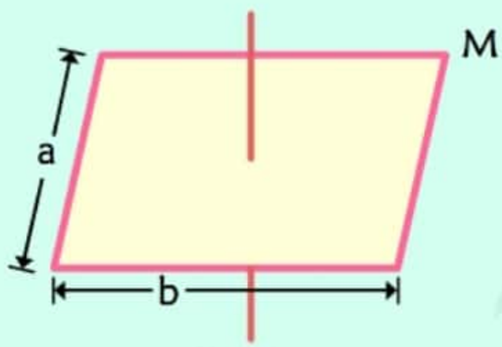
$$I = I_{com} + Md^2$$

Rod



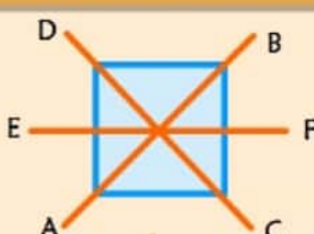
$\frac{ML^2}{3}$ (Uniform) $\frac{ML^2}{12}$ (Uniform)

Rectangular Plate



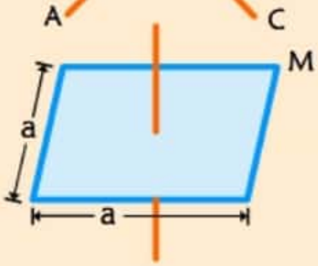
$$I = \frac{M(a^2+b^2)}{12} \text{ (Uniform)}$$

Square Plate



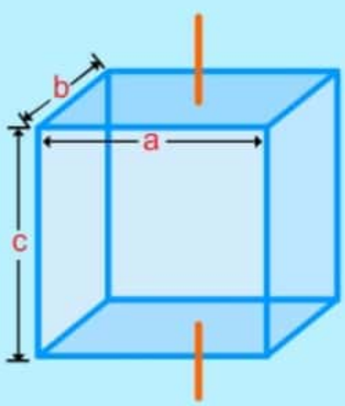
$$I_{AB} = I_{CD} = I_{EF}$$

$$= \frac{Ma^2}{12} \text{ (Uniform)}$$



$$\frac{Ma^2}{6} \text{ (Uniform)}$$

Cuboid



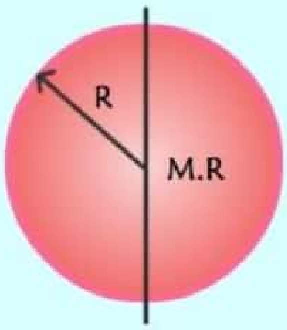
$$\frac{M(a^2+b^2)}{12}$$

(Uniform)

MOMENT OF INERTIA

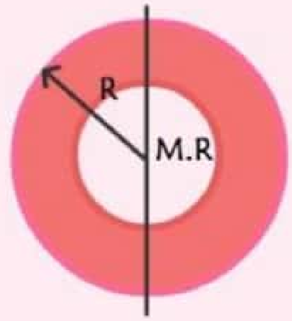
Part II

Solid Sphere



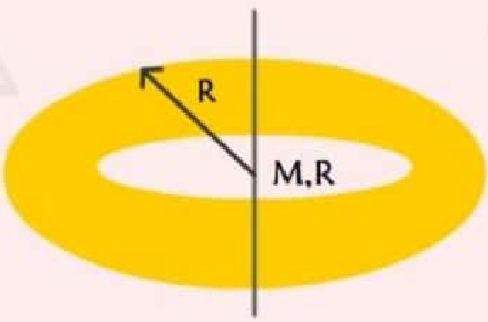
$I = \frac{2}{5} MR^2$ (Uniform)

Hollow Sphere



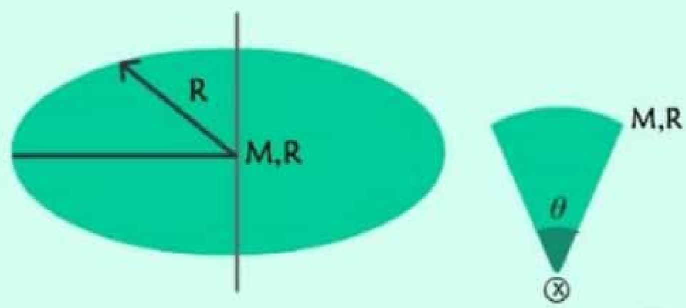
$I = \frac{2}{3} MR^2$ (Uniform)

Ring



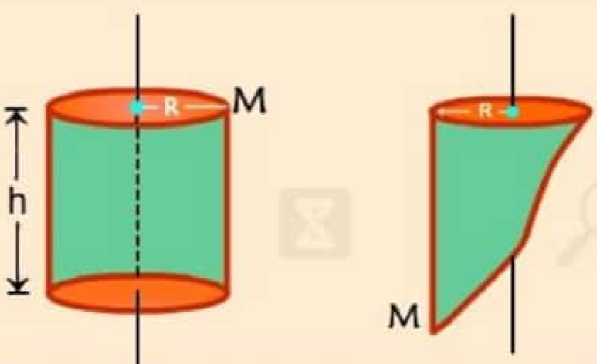
$I = MR^2$ (Uniform or Non Uniform)

Disc



$I = \frac{MR^2}{2}$ (Uniform)

Hollow cylinder



$I = MR^2$ (Uniform or Non Uniform)

Solid cylinder

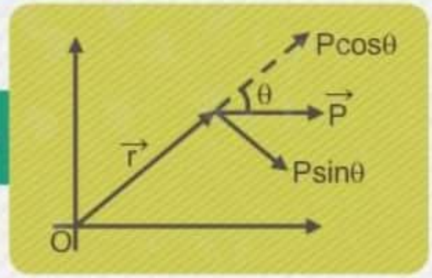
$I = \frac{MR^2}{2}$ (Uniform)

ANGULAR MOMENTUM

PART I

1 ANGULAR MOMENTUM OF A PARTICLE ABOUT A POINT

$\vec{L} = \vec{r} \times \vec{P} \Rightarrow L = rP\sin\theta$



2 ANGULAR MOMENTUM OF A RIGID BODY ROTATING ABOUT A FIXED AXIS

$L = I\omega$

Here, I is the moment of inertia of the rigid body about axis.

3 CONSERVATION OF ANGULAR MOMENTUM

The law of conservation of angular momentum states that when no external torque acts on an object, no change of angular momentum will occur.

Since $\vec{\tau}_{net} = \frac{d\vec{L}}{dt}$. Now if, $\vec{\tau}_{net} = 0$, then $\frac{d\vec{L}}{dt} = 0$, so that $\vec{L}$ = constant.

4 ANGULAR IMPULSE

The angular impulse of a torque in a given time interval is defined as $\int_{t_1}^{t_2} \vec{\tau}\,dt$

$\vec{\tau} = \frac{d\vec{L}}{dt}$

UNIFORM PURE ROLLING

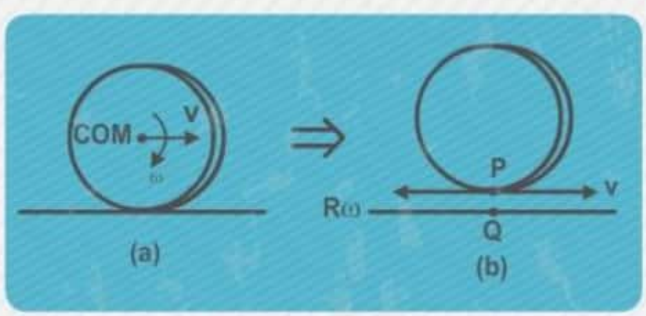
Pure rolling means no relative motion (or no slipping at point of contact between two bodies.)

$V_P = V_Q$ or $V - R\omega = 0$ or $V = R\omega$

If $V_P > V_Q$ or $V > R\omega$, the motion is said to be forward slipping and if

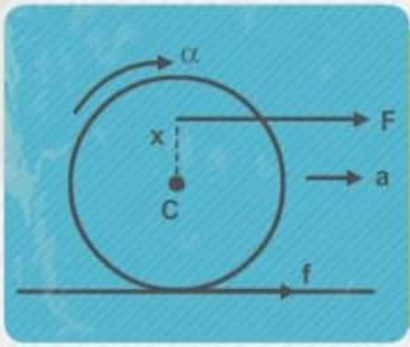
$V_P < V_Q < R\omega$, the motion is said to be backward slipping.

The condition of pure rolling on a stationary ground is, $a = R\alpha$



1 PURE ROLLING WHEN FORCE F ACT ON A BODY

Suppose a force F is applied at a distance x above the centre of a rigid body of radius R, mass M and moment of inertia CMR^2 about an axis passing through the centre of mass. Applied force F can produces by itself a linear acceleration a and an angular acceleration α.



a = liner acceleration , α = angular acceleration from linear motion

$$F + f = Ma$$

From rotational motion : $Fx - fR = I a$

$$a = \frac{F(R+X)}{MR(C+1)}, \quad f = \frac{F(x - RC)}{R(C+1)}$$

2 PURE ROLLING ON A INCLINED PLANS

A rigid body of radius R, and mass m is released at rest from height h on the incline whose inclination with horizontal is θ and assume that friciton is sufficient for pure rolling then,

$a = \alpha R$ and $v = R\omega$

ω = Angular Velocity

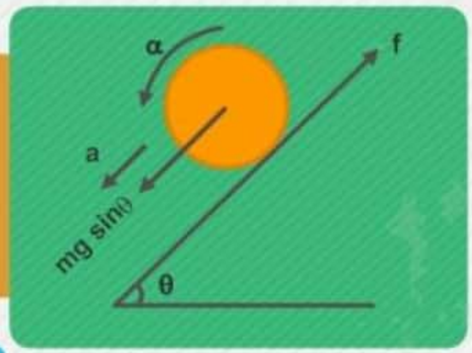
α = Angular Acceleration

Linear Acceleration,

$$a = \frac{g \sin\theta}{1 + C}$$

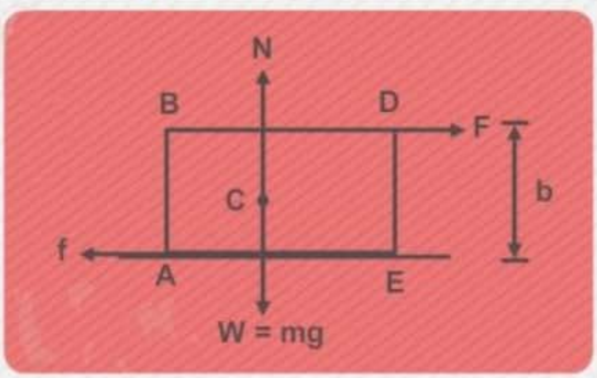
C = Center of Mass

So, body which have low value of C have greater acceleration.



TOPPLING

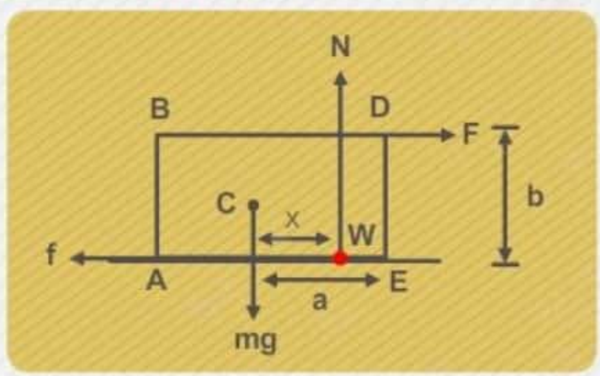
Torque about E



Balancing Torque at E

$$Fb = (mg)a \implies a = \frac{Fb}{mg}$$

Torque about W



Balancing Torque at W

$Fb + N(a - x) = mg\, a$

if $x = a$

$$F_{max} b = mga \implies F_{max} = \frac{mga}{b}$$