3

Motion in a Plane

Multiple Choice Questions (MCQs)

Q. 1 The angle between $\mathbf{A} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$ and $\mathbf{B} = \hat{\mathbf{i}} - \hat{\mathbf{j}}$ is

(a) 45° (b) 90° (c) – 45° (d) 180°

Thinking Process

To solve such type of questions, we have to use the formula for dot product or cross product.

Ans. *(b)* Given, $\mathbf{A} = \hat{i} + \hat{j}$

$\mathbf{B} = \hat{i} - \hat{j}$

We know that

$$\mathbf{A} \cdot \mathbf{B} = |\mathbf{A}||\mathbf{B}| \cos \theta$$

$$\Rightarrow \quad (\hat{i} + \hat{j}) \cdot (\hat{i} - \hat{j}) = (\sqrt{1+1})(\sqrt{1+1}) \cos \theta$$

where θ is the angle between **A** and **B**

$$\Rightarrow \quad \cos \theta = \frac{1 - 0 + 0 - 1}{\sqrt{2}\,\sqrt{2}} = \frac{0}{2} = 0$$

$$\Rightarrow \quad \theta = 90°$$

Q. 2 Which one of the following statements is true?

(a) A scalar quantity is the one that is conserved in a process

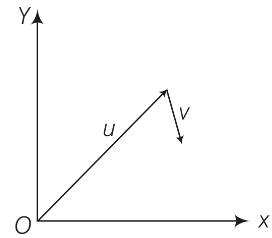
(b) A scalar quantity is the one that can never take negative values

(c) A scalar quantity is the one that does not vary from one point to another in space

(d) A scalar quantity has the same value for observers with different orientation of the axes

Ans. *(d)* A scalar quantity is independent of direction hence has the same value for observers with different orientations of the axes.

Q. 3 Figure shows the orientation of two vectors **u** and **v** in the *xy*-plane.



If $\mathbf{u} = a\hat{i} + b\hat{j}$ and $\mathbf{v} = p\hat{i} + q\hat{j}$

Which of the following is correct?

(a) *a* and *p* are positive while *b* and *q* are negative

(b) *a*, *p* and *b* are positive while *q* is negative

(c) *a*, *q* and *b* are positive while *p* is negative

(d) *a*, *b*, *p* and *q* are all positive

Thinking Process

In this question according to the diagram, we have to decide the components of a given vector.

Ans. *(b)* Clearly from the diagram, $\mathbf{u} = a\hat{i} + b\hat{j}$

As **u** is in the first quadrant, hence both components *a* and *b* will be positive.

For $\mathbf{v} = p\hat{i} + q\hat{j}$, as it is in positive *x*-direction and located downward hence *x*-component *p* will be positive and *y*-component *q* will be negative.

Q. 4 The component of a vector **r** along *X*-axis will have maximum value if

(a) **r** is along positive *Y*-axis

(b) **r** is along positive *X*-axis

(c) **r** makes an angle of 45° with the *X*-axis

(d) **r** is along negative *Y*-axis

Ans. *(b)* Let **r** makes an angle θ with positive *x*-axis component of **r** along *X*-axis

$$r_x = |\mathbf{r}| \cos\theta$$

$$(r_x)_{\text{maximum}} = |\mathbf{r}| (\cos\theta)_{\text{maximum}}$$

$$= |\mathbf{r}| \cos 0° = |\mathbf{r}| \qquad (\because \cos\theta \text{ is maximum of } \theta = 0°)$$

As $\theta = 0°$,

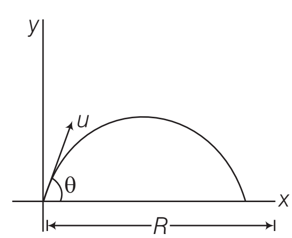
r is along positive *x*-axis.

Q. 5 The horizontal range of a projectile fired at an angle of 15° is 50 m. If it is fired with the same speed at an angle of 45°, its range will be

(a) 60 m (b) 71 m

(c) 100 m (d) 141 m

Ans. *(c)* We know that



where θ is angle of projection

Given, $\theta = 15°$ and $R = 50\,\text{m}$

$$\text{Range, } R = \frac{u^2 \sin 2\theta}{g}$$

Putting all the given values in the formula, we get

$$\Rightarrow \quad R = 50\,\text{m} = \frac{u^2 \sin(2 \times 15°)}{g}$$

$$\Rightarrow \quad 50 \times g = u^2 \sin 30° = u^2 \times \frac{1}{2}$$

$$\Rightarrow \quad 50 \times g \times 2 = u^2$$
$$\Rightarrow \quad u^2 = 50 \times 9.8 \times 2 = 100 \times 9.8 = 980$$
$$\Rightarrow \quad u = \sqrt{980} = \sqrt{49 \times 20} = 7 \times 2 \times \sqrt{5} \text{ m/s}$$
$$= 14 \times 2.23 \text{ m/s} = 31.304 \text{ m/s}$$
For $$\theta = 45°; R = \frac{u^2 \sin 2 \times 45°}{g} = \frac{u^2}{g} \qquad (\because \sin 90° = 1)$$
$$\Rightarrow \quad R = \frac{(14\sqrt{5})^2}{g} = \frac{14 \times 14 \times 5}{9.8} = 100 \text{ m}$$

Q. 6 Consider the quantities, pressure, power, energy, impulse, gravitational potential, electrical charge, temperature, area. Out of these, the only vector quantities are

(a) impulse, pressure and area
(b) impulse and area
(c) area and gravitational potential
(d) impulse and pressure

Ans. *(b)* We know that impulse $J = F.\,\Delta t = \Delta p$, where F is force, Δt is time duration and Δp is change in momentum. As Δp is a vector quantity, hence impulse is also a vector quantity. Sometimes area can also be treated as vector.

Q. 7 In a two dimensional motion, instantaneous speed v_0 is a positive constant. Then, which of the following are necessarily true?

(a) The average velocity is not zero at any time
(b) Average acceleration must always vanish
(c) Displacements in equal time intervals are equal
(d) Equal path lengths are traversed in equal intervals

Thinking Process

As speed is a scalar quantity, hence it will be related with path length (scalar quantity) only.

Ans. *(d)* We know that

$$\text{speed, } v_0 = \frac{\text{total distance travelled}}{\text{time taken}}$$

Hence, total distance travelled = Path length
= (speed) × time taken

Note *We should be very careful with the fact, that speed is related with total distance covered not with displacement.*

Q. 8 In a two dimensional motion, instantaneous speed v_0 is a positive constant. Then, which of the following are necessarily true?

(a) The acceleration of the particle is zero
(b) The acceleration of the particle is bounded
(c) The acceleration of the particle is necessarily in the plane of motion
(d) The particle must be undergoing a uniform circular motion

Ans. *(c)* As given motion is two dimensional motion and given that instantaneous speed v_0 is positive constant. Acceleration is rate of change of velocity (instantaneous speed) hence it will also be in the plane of motion.

Multiple Choice Questions (More Than One Options)

Q. 9 Three vectors **A**, **B** and **C** add upto zero. Find which is false.

(a) $(\mathbf{A} \times \mathbf{B}) \times \mathbf{C}$ is not zero unless **B, C** are parallel

(b) $(\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C}$ is not zero unless **B, C** are parallel

(c) If **A, B, C** define a plane, $(\mathbf{A} \times \mathbf{B}) \times \mathbf{C}$ is in that plane

(d) $(\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C} = |\mathbf{A}|\,|\mathbf{B}||\mathbf{C}| \rightarrow \mathbf{C}^2 = \mathbf{A}^2 + \mathbf{B}^2$

Thinking Process

This question can solved by checking each options one by one.

Ans. *(b, d)*

Given A + B + C = 0

Hence, we can say that A, B and C are in one plane and are represented by the three sides of a triangle taken in one order. Now consider the options one by one.

(a) We can write

$$\mathbf{B} \times (\mathbf{A} + \mathbf{B} + \mathbf{C}) = \mathbf{B} \times 0 = 0$$

$\Rightarrow$ $$\mathbf{B} \times \mathbf{A} + \mathbf{B} \times \mathbf{B} + \mathbf{B} \times \mathbf{C} = 0$$

$\Rightarrow$ $$\mathbf{B} \times \mathbf{A} + 0 + \mathbf{B} \times \mathbf{C} = 0$$

$\Rightarrow$ $$\mathbf{B} \times \mathbf{A} = -\mathbf{B} \times \mathbf{C}$$

$\Rightarrow$ $$\mathbf{A} \times \mathbf{B} = \mathbf{B} \times \mathbf{C}$$

$\therefore$ $$(\mathbf{A} \times \mathbf{B}) \times \mathbf{C} = (\mathbf{B} \times \mathbf{C}) \times \mathbf{C}$$

It cannot be zero.

If $\mathbf{B} \,||\, \mathbf{C}$, then $\mathbf{B} \times \mathbf{C} = 0$, than $(\mathbf{B} \times \mathbf{C}) \times \mathbf{C} = \mathbf{0}$.

(b) $(\mathbf{A} \times \mathbf{B}) . \mathbf{C} = (\mathbf{B} \times \mathbf{AC}) . \mathbf{C} = 0$ whatever be the positions of A, B and C. If $\mathbf{B} \,||\, \mathbf{C}$, then $\mathbf{B} \times \mathbf{C} = 0$, then $(\mathbf{B} \times \mathbf{C}) \times \mathbf{C} = 0$.

(c) $(\mathbf{A} \times \mathbf{B}) = \mathbf{X} = \mathbf{AB} \sin\theta\, \mathbf{X}$. The direction of X is perpendicular to the plane containing A and B. $(\mathbf{A} \times \mathbf{B}) \times \mathbf{C} = \mathbf{X} \times \mathbf{C}$. Its direction is in the plane of A, B and C.

(d) If $\mathbf{C}^2 = \mathbf{A}^2 + \mathbf{B}^2$, then angle between **A** and **B** is 90°

$\therefore$ $$(\mathbf{A} \times \mathbf{B}) . \mathbf{C} = (AB \sin 90^\circ\, \mathbf{X}) . \mathbf{C} = AB\,(\mathbf{X} . \mathbf{C})$$
$$= ABC \cos 90^\circ = 0$$

Q. 10 It is found that $|\mathbf{A} + \mathbf{B}| = |\mathbf{A}|$. This necessarily implies.

(a) $\mathbf{B} = 0$

(b) **A**, **B** are antiparallel

(c) **A**, **B** are perpendicular

(d) $\mathbf{A} . \mathbf{B} \leq 0$

Ans. *(a, b)*

Given that $$|\mathbf{A} + \mathbf{B}| = |\mathbf{A}| \text{ or } |\mathbf{A} + \mathbf{B}|^2 = |\mathbf{A}|^2$$

$\Rightarrow$ $$|\mathbf{A}|^2 + |\mathbf{B}|^2 + 2|\mathbf{A}||\mathbf{B}|\cos\theta = |\mathbf{A}|^2$$

where θ is angle between **A** and **B**.

$\Rightarrow$ $$|\mathbf{B}|\,(|\mathbf{B}| + 2\,|\mathbf{A}|\cos\theta) = 0$$

$\Rightarrow$ $$|\mathbf{B}| = 0 \text{ or } |\mathbf{B}| + 2|\mathbf{A}|\cos\theta = 0$$

$\Rightarrow$ $$\cos\theta = -\frac{|\mathbf{B}|}{2|\mathbf{A}|} \quad \text{...(i)}$$

If **A** and **B** are antiparallel, then $\theta = 180°$

Hence, from Eq. (i)

$$-1 = -\frac{|\mathbf{B}|}{2|\mathbf{A}|} \Rightarrow |\mathbf{B}| = 2|\mathbf{A}|$$

Hence, correct answer will be either $|\mathbf{B}| = 0$ or **A** and **B** are antiparallel provided $|\mathbf{B}| = 2\,|\mathbf{A}|$

Q. 11 Two particles are projected in air with speed v_0 at angles θ_1 and θ_2 (both acute) to the horizontal, respectively. If the height reached by the first particle is greater than that of the second, then tick the right choices.

(a) Angle of projection $q_1 > q_2$

(b) Time of flight $T_1 > T_2$

(c) Horizontal range $R_1 > R_2$

(d) Total energy $U_1 > U_2$

Thinking Process

In this problem, we have to apply equation for maximum height reached $H = \frac{u^2 \sin^2\theta}{2g}$, where θ is angle of projection and u is speed of projection of a projectile motion.

Ans. ***(a, b, c)***

We know that maximum height reached by a projectile,

$$H = \frac{u^2 \sin^2 \theta}{2g}$$

$$H_1 = \frac{v_0^2 \sin^2 \theta_1}{2g} \quad \text{(for first particle)}$$

$$H_2 = \frac{v_0^2 \sin^2 \theta_2}{2g} \quad \text{(for second particle)}$$

According to question, we know that

$$H_1 > H_2$$

$$\Rightarrow \quad \frac{v_0^2 \sin^2 \theta_1}{2g} > \frac{v_0^2 \sin^2 \theta_2}{2g}$$

$$\Rightarrow \quad \sin^2 \theta_1 > \sin^2 \theta_2$$

$$\Rightarrow \quad \sin^2 \theta_1 - \sin^2 \theta_2 > 0$$

$$\Rightarrow \quad (\sin\theta_1 - \sin\theta_2)(\sin\theta_1 + \sin\theta_2) > 0$$

Thus, either $\sin\theta_1 + \sin\theta_2 > 0$

$$\Rightarrow \quad \sin\theta_1 - \sin\theta_2 > 0$$

$$\Rightarrow \quad \sin\theta_1 > \sin\theta_2 \text{ or } \theta_1 > \theta_2$$

Time of fight, $$T = \frac{2u\sin\theta}{g} = \frac{2\,v_0 \sin\theta}{g}$$

Thus, $$T_1 = \frac{2\,v_0 \sin\theta_1}{g}$$

$$T_2 = \frac{2\,v_0 \sin\theta_2}{g}$$

(Here, T_1 = Time of flight of first particle and T_2 = Time of flight of second particle).

As, $\sin\theta_1 > \sin\theta_2$

Hence, $T_1 > T_2$

We know that

$$\text{Range, } R = \frac{u^2 \sin 2\theta}{g} = \frac{v_0^2 \sin 2\theta}{g}$$

$$R_1 = \text{Range of first particle} = \frac{u_0^2 \sin 2\,\theta_1}{g}$$

$$R_2 = \text{Range of second particle} = \frac{v_0^2 \sin 2\theta_2}{g}$$

Given, $\sin\theta_1 > \sin\theta_2$

$\Rightarrow$ $\sin 2\,\theta_1 > \sin 2\,\theta_2$

$\Rightarrow$ $\dfrac{R_1}{R_2} = \dfrac{\sin 2\,\theta_1}{\sin 2\,\theta_2} > 1$

$\Rightarrow$ $R_1 > R_2$

Total energy for the first particle,

$$U_1 = \text{KE} + \text{PE} = \frac{1}{2} m_1 v_0^2$$

(This value will be constant throughout the journey)

$$U_2 = \text{KE} + \text{PE} = \frac{1}{2} m_2 v_0^2 \quad \text{(Total energy for the second particle)}$$

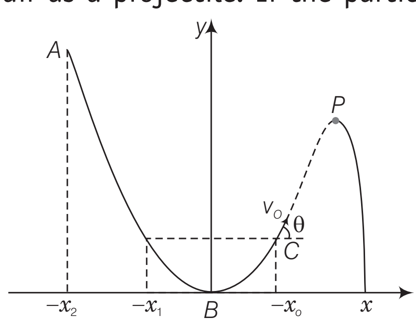
Total energy for the second particle

If $m_1 = m_2$ then $U_1 = U_2$

$m_1 > m_2$ then $U_1 > U_2$

$m_1 < m_2$, then $U_1 < U_2$

Q. 12 A particle slides down a frictionless parabolic $(y + x^2)$ track $(A - B - C)$ starting from rest at point A (figure). Point B is at the vertex of parabola and point C is at a height less than that of point A. After C, the particle moves freely in air as a projectile. If the particle reaches highest point at P, then



$(x = 0)$

(a) KE at P = KE at B

(b) height at P = height at A

(c) total energy at P = total energy at A

(d) time of travel from A to B = time of travel from B to P

Thinking Process

In this type of question, nature of track is very important of consider, as friction is not in this track, total energy of the particle will remain constant throughout the journey.

Ans. ***(c)***

As the given track $y = x^2$ is a frictionless track thus, total energy (KE + PE) will be same throughout the journey.

Hence, total energy at A = Total energy at P. At B, the particle is having only KE but at P some KE is converted to P.

Hence, $$(KE)_B > (KE)_P$$

Total energy at A = PE = Total energy at B = KE

= Total energy at P

= PE + KE

The potential energy at A, is converted to KE and PE at P, hence

$$(PE)\,P < (PE)\,A$$

Hence, $$(\text{Height})\,P < (\text{Height})\,A$$

As, height of P < Height of A

Hence, path length AB > path length BP

Hence, time of travel from A to B ≠ Time of travel from B to P.

Q. 13 **Following are four different relations about displacement, velocity and acceleration for the motion of a particle in general. Choose the incorrect one (s).**

(a) $v_{av} = \frac{1}{2}[v(t_1) + v(t_2)]$

(b) $v_{av} = \frac{r(t_2) - r(t_1)}{t_2 - t_1}$

(c) $r = \frac{1}{2}(v(t_2) - v(t_1))(t_2 - t_1)$

(d) $a_{av} = \frac{v(t_2) - v(t_1)}{t_2 - t_1}$

Ans. ***(a, c)***

If an object undergoes a displacement Δr in time Δt, its average velocity is given by

$v = \frac{\Delta r}{\Delta t} = \frac{r_2 - r_1}{t_2 - t_1}$; where r_1 and r_2 are position vectors corresponding to time t_1 and t_2.

It the velocity of an object changes from v_1 to v_2 in time Δt. Average acceleration is given by

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_2}$$

But, when acceleration is non-uniform

$$v_{av} \neq \frac{v_1 + v_2}{2}$$

We can write $$\Delta v = \frac{\Delta r}{\Delta t}$$

Hence, $$\Delta r = r_2 - r_1 = (v_2 - v_1)(t_2 - t_1)$$

Q. 14 **For a particle performing uniform circular motion, choose the correct statement(s) from the following.**

(a) Magnitude of particle velocity (speed) remains constant

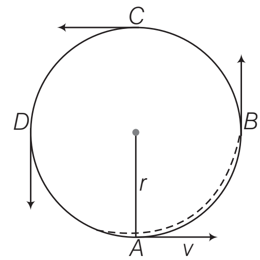
(b) Particle velocity remains directed perpendicular to radius vector

(c) Direction of acceleration keeps changing as particle moves

(d) Angular momentum is constant in magnitude but direction keeps changing

Ans. ***(a, b, c)***

For a particle performing uniform circular motion



(i) speed will be constant throughout.

(ii) velocity will be tangential in the direction of motion at a particular point.

(iii) acceleration $a = \frac{v^2}{r}$, will always be towards centre of the circular path.

(iv) angular momentum (mvr) is constant in magnitude and direction/out of the plane perpendicularly, as well.

Note *In uniform circular motion, magnitude of velocity and acceleration is constant but direction changes continuously.*

Q. 15 For two vectors **A** and **B**, $|\mathbf{A}+\mathbf{B}| = |\mathbf{A}-\mathbf{B}|$ is always true when

(a) $|\mathbf{A}| = |\mathbf{B}| \neq 0$

(b) $\mathbf{A} \perp \mathbf{B}$

(c) $|\mathbf{A}| = |\mathbf{B}| \neq 0$ and **A** and **B** are parallel or anti-parallel

(d) when either $|\mathbf{A}|$ or $|\mathbf{B}|$ is zero

Ans. ***(b, d)***

Given, $|\mathbf{A}+\mathbf{B}| = |\mathbf{A}-\mathbf{B}|$

$$\Rightarrow \sqrt{|\mathbf{A}|^2 + |\mathbf{B}|^2 + 2|\mathbf{A}||\mathbf{B}|\cos\theta} = \sqrt{|\mathbf{A}|^2 + |\mathbf{B}|^2 - 2|\mathbf{A}||\mathbf{B}|\cos\theta}$$

$$\Rightarrow |\mathbf{A}|^2 + |\mathbf{B}|^2 + 2|\mathbf{A}||\mathbf{B}|\cos\theta = |\mathbf{A}|^2 + |\mathbf{B}|^2 - 2|\mathbf{A}||\mathbf{B}|\cos\theta$$

$$\Rightarrow 4|\mathbf{A}||\mathbf{B}|\cos\theta = 0$$

$$\Rightarrow |\mathbf{A}||\mathbf{B}|\cos\theta = 0$$

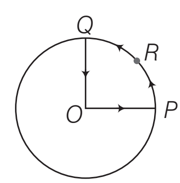
$$\Rightarrow |\mathbf{A}| = 0 \text{ or } |\mathbf{B}| = 0 \text{ or } \cos\theta = 0$$

$$\Rightarrow \theta = 90^\circ$$

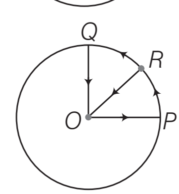
When $\theta = 90^\circ$, we can say that $\mathbf{A} \perp \mathbf{B}$

Very Short Answer Type Questions

Q. 16 A cyclist starts from centre O of a circular park of radius 1 km and moves along the path $OPRQO$ as shown in figure. If he maintains constant speed of 10 ms^{-1}, what is his acceleration at point R in magnitude and direction?



Ans. As shown in the adjacent figure. The cyclist covers the path $OPRQO$.

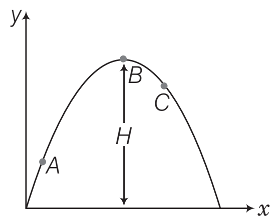


As we know whenever an object performing circular motion, acceleration is called centripetal acceleration and is always directed towards the centre.

Hence, acceleration at R is $a = \frac{v^2}{r}$

$$\Rightarrow a = \frac{(10)^2}{1\,\text{km}} = \frac{100}{10^3} = 0.1\,\text{m/s}^2 \text{ along } RO.$$

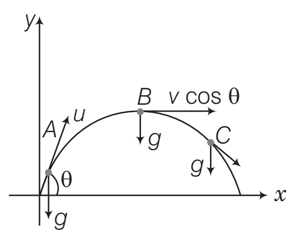
Q. 17 A particle is projected in air at some angle to the horizontal, moves along parabola as shown in figure where x and y indicate horizontal and vertical directions, respectively. Shown in the diagram, direction of velocity and acceleration at points A, B and C.



Thinking Process

When a particle is under projectile motion, horizontal component of velocity will always be constant and acceleration is always vertically downward and is equal to g.

Ans. Consider the adjacent diagram in which a particle is projected at an angle θ.



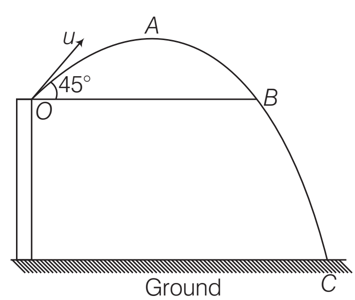
v_x = Horizontal component of velocity $= v\cos\theta$ = constant.
v_y = Vertical component of velocity $= v\sin\theta$
Velocity will always be tangential to the curve in the direction of motion and acceleration is always vertically downward and is equal to g (acceleration due to gravity).

Q. 18 A ball is thrown from a roof top at an angle of 45° above the horizontal. It hits the ground a few seconds later. At what point during its motion, does the ball have

(a) greatest speed
(b) smallest speed
(c) greatest acceleration

Explain.

Ans. Consider the adjacent diagram in which a ball is projected from point O, and covering the path $OABC$.



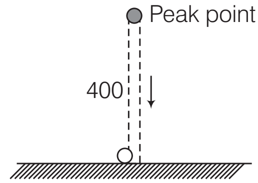
(a) At point B, it will gain the same speed u and after that speed increases and will be maximum just before reaching C.

(b) During upward journey from O to A speed decreases and will be minimum at point A.

(c) Acceleration is always constant throughout the journey and is vertically downward equal to g.

Q. 19 **A football is kicked into the air vertically upwards. What is its (a) acceleration and (b) velocity at the highest point?**

Ans. **(a)** Consider the adjacent diagram in which a football is kicked into the air vertically upwards. Acceleration of the football will always be vertical downward and is equal to g.



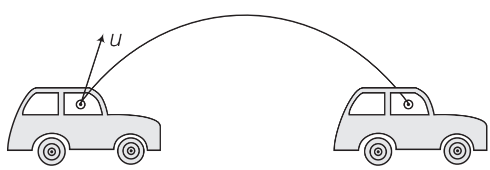
(b) When the football reaches the highest point velocity will be zero as it is continuously retarded by acceleration due to gravity g.

Q. 20 **A, B and C are three non-collinear, non co-planar vectors. What can you say about direction of $\mathbf{A}\times(\mathbf{B}\times\mathbf{C})$?**

Ans. The direction of $(\mathbf{B}\times\mathbf{C})$ will be perpendicular to the plane containing **B** and **B** by right hand rule. $\mathbf{A}\times(\mathbf{B}\times\mathbf{C})$ will lie in the plane of **B** and **C** and is perpendicular to vector **A**.

Q. 21 **A boy travelling in an open car moving on a levelled road with constant speed tosses a ball vertically up in the air and catches it back. Sketch the motion of the ball as observed by a boy standing on the footpath. Give explanation to support your diagram.**

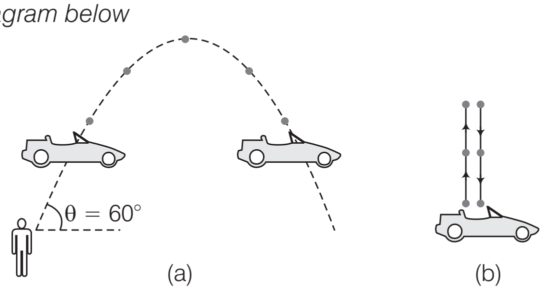
Ans. The path of the ball observed by a boy standing on the footpath is parabolic. The horizontal speed of the ball is same as that of the car, therefore, ball as well car travells equal horizontal distance. Due to its vertical speed, the ball follows a parabolic path.



Note *We must be very clear that we are working with respect to ground. When we observe with respect to the car motion will be along vertical direction only.*

Q. 22 **A boy throws a ball in air at 60° to the horizontal along a road with a speed of 10 m/s (36 km/h). Another boy sitting in a passing by car observes the ball. Sketch the motion of the ball as observed by the boy in the car, if car has a speed of (18km/h). Give explanation to support your diagram.**

Ans. *Consider the diagram below*



The boy throws the ball at an angle of 60°.

$\therefore$ Horizontal component of velocity = $4\cos\theta$

$$= (10 \text{ m/s}) \cos 60° = 10 \times \frac{1}{2} = 5 \text{ m/s}.$$

Speed of the car = 18 km/h = 5 m/s.

As horizontal speed of ball and car is same, hence relative velocity of car and ball in the horizontal direction will be zero.

Only vertical motion of the ball will be seen by the boy in the car, as shown in fig. (b)

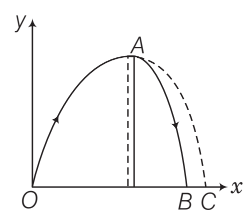
Q. 23 **In dealing with motion of projectile in air, we ignore effect of air resistance on motion. This gives trajectory as a parabola as you have studied. What would the trajectory look like if air resistance tance is include? Sketch such a trajectory and explain why you have drawn it that way.**

Thinking Process

When air resistance is included the horizontal component of velocity will not be constant and obviously trajectory will change.

Ans. Due to air resistance, particle energy as well as horizontal component of velocity keep on decreasing making the fall steeper than rise as shown in the figure.

When we are neglecting air resistance path was symmetric parabola (*OAB*). When air resistance is considered path is asymmetric parabola (*OAC*).



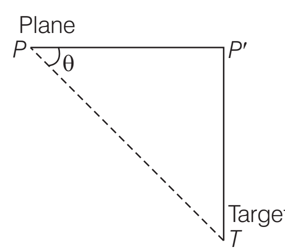
Short Answer Type Questions

Q. 24 **A fighter plane is flying horizontally at an altitude of 1.5 km with speed 720 km/h. At what angle of sight (w.r.t. horizontal) when the target is seen, should the pilot drop the bomb in order to attack the target?**

Thinking Process

When the bomb is dropped from the plane, the bomb will have same velocity as that of plane.

Ans. Consider the adjacent diagram. Let a fighter plane, when it be at position *P*, drops a bomb to hit a target *T*.



Let $\angle P'PT = \theta$

Speed of the plane = $720 \text{ km/h} = 720 \times \frac{5}{18} \text{ m/s} = 200 \text{ m/s}$

Altitude of the plane ($P'T$) = 1.5 km = 1500 m

If bomb hits the target after time t, then horizontal distance travelled by the bomb,

$$PP' = u \times t = 200t \quad \text{...(i)}$$

Vertical distance travelled by the bomb,

$$P'T = \frac{1}{2}gt^2 \Rightarrow 1500 = \frac{1}{2} \times 9.8t^2$$

$$\Rightarrow \quad t^2 = \frac{1500}{4.9} \Rightarrow t = \sqrt{\frac{1500}{49}} = 17.49 \text{ s}$$

Using value of t in Eq. (i),

$$PP' = 200 \times 17.49 \text{ m}$$

Now, $$\tan\theta = \frac{P'T}{P'P} = \frac{1500}{200 \times 17.49} = .49287 = \tan 23° 12'$$

$$\theta = 23° 12'$$

Note *Angle is with respect to target. As seen by observer in the plane motion of the bomb will be vertically downward below the plane.*

Q. 25 (a) Earth can be thought of as a sphere of radius 6400 km. Any object (or a person) is performing circular motion around the axis of the earth due to the earth rotation (period 1 day). What is acceleration of object on the surface of the earth (at equator) towards its centre? What is it at latitude θ? How does these accelerations compare with $g = 9.8 \text{ m/s}^2$?

(b) Earth also moves in circular orbit around the sun once every year with an orbital radius of 1.5×10^{11} m. What is the acceleration of the earth (or any object on the surface of the earth) towards the centre of the sun? How does this acceleration compare with $g = 9.8 \text{ m/s}^2$?

Ans. **(a)** Radius of the earth $(R) = 6400 \text{ km} = 6.4 \times 10^6 \text{ m}$

Time period $(T) = 1 \text{ day} = 24 \times 60 \times 60 \, s = 86400 \text{ s}$

Centripetal acceleration $(a_c) = \omega^2 R = R\left(\frac{2\pi}{T}\right)^2 = \frac{4\pi^2 R}{T}$

$$= \frac{4 \times (22/7)^2 \times 6.4 \times 10^6}{(24 \times 60 \times 60)^2}$$

$$= \frac{4 \times 484 \times 64 \times 10^6}{49 \times (24 \times 3600)^2}$$

$$= 0.034 \text{ m/s}^2$$

At equator, latitude $\theta = 0°$

$$\therefore \quad \frac{a_c}{g} = \frac{0.034}{9.8} = \frac{1}{288}$$

(b) Orbital radius of the earth around the sun $(R) = 1.5 \times 10^{11}$ m

Time period = 1 yr = 365 day

$$= 365 \times 24 \times 60 \times 60 \, s = 3.15 \times 10^7 \, s$$

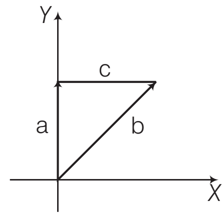
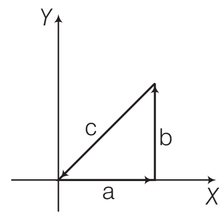
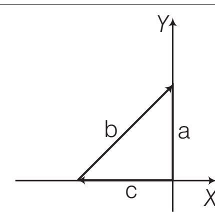
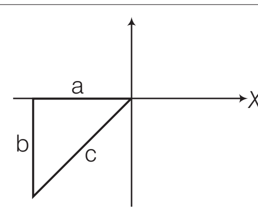
Centripetal acceleration $(a_c) = R\omega^2 = \frac{4\pi^2 R}{T^2}$

$$= \frac{4 \times (22/7)^2 \times 1.5 \times 10^{11}}{(3.15 \times 10^7)^2}$$

$$= 5.97 \times 10^{-3} \text{ m/s}^2$$

$$\therefore \quad \frac{a_c}{g} = \frac{5.97 \times 10^{-3}}{9.8} = \frac{1}{1642}$$

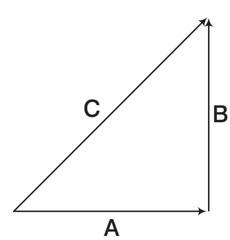
Q. 26 Given below in Column I are the relations between vectors **a**, **b** and **c** and in Column II are the orientations of **a**, **b** and **c** in the *XY*- plane. Match the relation in Column I to correct orientations in Column II.

Column I	Column II
(a) $\mathbf{a} + \mathbf{b} = \mathbf{c}$	(i) 
(b) $\mathbf{a} - \mathbf{c} = \mathbf{b}$	(ii) 
(c) $\mathbf{b} - \mathbf{a} = \mathbf{c}$	(iii) 
(d) $\mathbf{a} + \mathbf{b} + \mathbf{c} = 0$	(iv) 

Thinking Process

In this problem, triangular law of vector addition will be applied.

Ans. Consider the adjacent diagram in which vectors **A** and **B** are corrected by head and tail.



Resultant vector $\mathbf{C} = \mathbf{A} + \mathbf{B}$

(a) from (iv) it is clear that $\mathbf{c} = \mathbf{a} + \mathbf{b}$

(b) from (iii) $\mathbf{c} + \mathbf{b} = \mathbf{a} \Rightarrow \mathbf{a} - \mathbf{c} = \mathbf{b}$

(c) from (i) $\mathbf{b} = \mathbf{a} + \mathbf{c} \Rightarrow \mathbf{b} - \mathbf{a} = \mathbf{c}$

(d) from (ii) $-\mathbf{c} = \mathbf{a} + \mathbf{b} \Rightarrow \mathbf{a} + \mathbf{b} + \mathbf{c} = 0$

Q. 27 If $|\mathbf{A}| = 2$ and $|\mathbf{B}| = 4$, then match the relation in Column I with the angle θ between A and B in Column II.

	Column I		Column II
(a)	$\mathbf{A \cdot B} = 0$	(i)	$\theta = 0$
(b)	$\mathbf{A \cdot B} = +8$	(ii)	$\theta = 90°$
(c)	$\mathbf{A \cdot B} = 4$	(iii)	$\theta = 180°$
(d)	$\mathbf{A \cdot B} = -8$	(iv)	$\theta = 60°$

Ans. Given $|\mathbf{A}| = 2$ and $|\mathbf{B}| = 4$

(a) $\mathbf{A \cdot B} = AB\cos\theta = 0 \quad \Rightarrow \quad 2 \times 4\cos\theta = 0$

$\Rightarrow \quad \cos\theta = 0 = \cos 90° \quad \Rightarrow \quad \theta = 90°$

$\therefore$ Option (a) matches with option (ii).

(b) $\mathbf{A \cdot B} = AB\cos\theta = 8 \quad \Rightarrow \quad 2 \times 4\cos\theta = 8$

$\Rightarrow \quad \cos\theta = 1 = \cos 0° \quad \Rightarrow \quad \theta = 0°$

$\therefore$ Option (b) matches with option (i).

(c) $\mathbf{A \cdot B} = AB\cos\theta = 4 \quad \Rightarrow \quad 2 \times 4\cos\theta = 4$

$\Rightarrow \quad \cos\theta = \frac{1}{2} = \cos 60° \quad \Rightarrow \quad \theta = 60°$

$\therefore$ Option (c) matches with option (iv).

(d) $\mathbf{A \cdot B} = AB\cos\theta = -8 \quad \Rightarrow \quad 2 \times 4\cos\theta = -8$

$\Rightarrow \quad \cos\theta = -1 = \cos 180° \quad \Rightarrow \quad \theta = 180°$

$\therefore$ Option (d) matches with option (iii).

Q. 28 If $|\mathbf{A}| = 2$ and $|\mathbf{B}| = 4$, then match the relations in Column I with the angle θ between A and B in Column II

	Column I		Column II
(a)	$|\mathbf{A} \times \mathbf{B}| = 0$	(i)	$\theta = 30°$
(b)	$|\mathbf{A} \times \mathbf{B}| = 8$	(ii)	$\theta = 45°$
(c)	$|\mathbf{A} \times \mathbf{B}| = 4$	(iii)	$\theta = 90°$
(d)	$|\mathbf{A} \times \mathbf{B}| = 4\sqrt{2}$	(iv)	$\theta = 0°$

Ans. Given $|\mathbf{A}| = 2$ and $|\mathbf{B}| = 4$

(a) $|\mathbf{A} \times \mathbf{B}| = AB\sin\theta = 0 \quad \Rightarrow \quad 2 \times 4 \times \sin\theta = 0$

$\Rightarrow \quad \sin\theta = 0 = \sin 0° \quad \Rightarrow \quad \theta = 0°$

$\therefore$ Option (a) matches with option (iv).

(b) $|\mathbf{A} \times \mathbf{B}| = AB\sin\theta = 8 \quad \Rightarrow \quad 2 \times 4\sin\theta = 8$

$\Rightarrow \quad \sin\theta = 1 = \sin 90° \quad \Rightarrow \quad \theta = 90°$

$\therefore$ Option (b) matches with option (iii).

(c) $|\mathbf{A} \times \mathbf{B}| = AB\sin\theta = 4 \quad \Rightarrow \quad 2 \times 4\sin\theta = 4$

$\Rightarrow \quad \sin\theta = \frac{1}{2} = \sin 30° \quad \Rightarrow \quad \theta = 30°$

$\therefore$ Option (c) matches with option (i).

(d) $|\mathbf{A} \times \mathbf{B}| = AB\sin\theta = 4\sqrt{2} \quad \Rightarrow \quad 2 \times 4\sin\theta = 4\sqrt{2}$

$\Rightarrow \quad \sin\theta = \frac{1}{\sqrt{2}} = \sin 45° \quad \Rightarrow \quad \theta = 45°$

$\therefore$ Option (d) matches with option (ii).

Long Answer Type Questions

Q. 29 A hill is 500 m high. Supplies are to be sent across the hill using a canon that can hurl packets at a speed of 125 m/s over the hill. The canon is located at a distance of 800 m from the foot of hill and can be moved on the ground at a speed of 2 m/s; so that its distance from the hill can be adjusted. What is the shortest time in which a packet can reach on the ground across the hill? Take, $g = 10\ m/s^2$.

Ans. Given, speed of packets = 125 m/s

Height of the hill = 500 m.

To cross the hill, the vertical component of the velocity should be sufficient to cross such height.

$$u_y \geq \sqrt{2gh}$$
$$\geq \sqrt{2 \times 10 \times 500}$$
$$\geq 100\text{m/s}$$

But $$u^2 = u_x^2 + u_y^2$$

∴ Horizontal component of initial velocity,

$$u_x = \sqrt{u^2 - u_y^2} = \sqrt{(125)^2 - (100)^2} = 75\ m/s$$

Time taken to reach the top of the hill,

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 500}{10}} = 10\ s$$

Time taken to reach the ground from the top of the hill $t' = t = 10$ s. Horizontal distance travelled in 10 s

$$x = u_x \times t = 75 \times 10 = 750\ m$$

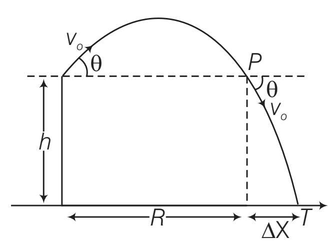
∴ Distance through which canon has to be moved = 800 − 750 = 50 m

Speed with which canon can move = 2 m/s

∴ Time taken by canon $= \frac{50}{2} \Rightarrow t'' = 25$ s

∴ Total time taken by a packet to reach on the ground $= t'' + t + t' = 25 + 10 + 10 = 45$ s

Q. 30 A gun can fire shells with maximum speed v_0 and the maximum horizontal range that can be achieved is $\mathbf{R} = \frac{v_0^2}{g}$. If a target farther away by distance Δx (beyond R) has to be hit with the same gun, show that it could be achieved by raising the gun to a height at least $h = \Delta x \left[1 + \frac{\Delta x}{R}\right]$



Thinking Process

Horizontal range of a projectile is maximum when it is thrown at an angle 45° from the horizontal and $R_{max} = \frac{u^2}{g}$, where u is speed of projection of the projectile.

Ans. *This problem can be approached in two different ways*

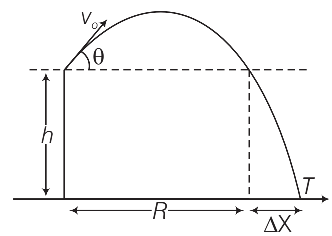
(i) Refer to the diagram, target T is at horizontal distance $x = R + \Delta x$ and between point of projection $y = -h$.

(ii) From point P in the diagram projection at speed v_0 at an angle θ below horizontal with height h and horizontal range ΔxA)

Applying method (i)

Maximum horizontal range

$$R = \frac{v_0^2}{g}, \text{ for } \theta = 45° \qquad \text{...(i)}$$



Let the gun be raised through a height h from the ground so that it can hit the target. Let vertically downward direction is taken as positive

Horizontal component of initial velocity $= v_0 \cos\theta$

Vertical component of initial velocity $= - v_0 \sin\theta$

Taking motion in vertical direction, $h = (- v_0 \sin\theta) t + \frac{1}{2} g t^2$ ...(ii)

Taking motion in horizontal direction

$$(R + \Delta x) = v_0 \cos\theta \times t$$

$$\Rightarrow \quad t = \frac{(R + \Delta x)}{v_0 \cos\theta} \qquad \text{...(iii)}$$

Substituting value of t in Eq. (ii), we get

$$h = (- v_0 \sin\theta) \times \left(\frac{R + \Delta x}{v_0 \cos\theta}\right) + \frac{1}{2} g \left(\frac{R + \Delta x}{v_0 \cos\theta}\right)^2$$

$$h = - (R + \Delta x) \tan\theta + \frac{1}{2} g \frac{(R + \Delta x)^2}{v_0^2 \cos^2 \theta}$$

As angle of projection is $\theta = 45°$, therefore

$$h = - (R + \Delta x) + \tan 45° + \frac{1}{2} g \frac{(R + \Delta x)^2}{v_0^2 \cos^2 45°}$$

$$h = - (R + \Delta x) \times 1 + \frac{1}{2} g \frac{(R + \Delta x)^2}{v_0^2 (1/2)}$$

$$\left(\because \tan 45° = 1 \text{ and } \cos 45° = \frac{1}{\sqrt{2}}\right)$$

$$h = - (R + \Delta x) + \frac{(R + \Delta x)^2}{R} \qquad [\text{Using Eq. (i)}, R = v_0^2 / g]$$

$$= - (R + \Delta x) + \frac{1}{R} (R^2 + \Delta x^2 + 2R\Delta x)$$

$$= - R - \Delta x + \left(R + \frac{\Delta x^2}{R} + 2\Delta x\right)$$

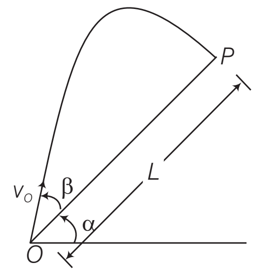
$$= \Delta x + \frac{\Delta x^2}{R}$$

$$h = \Delta x \left(1 + \frac{\Delta x}{R}\right)$$

Hence proved.

Note *We should not confuse with the positive direction of motion. May be vertically upward direction or vertically downward direction is taken as positive according to convenience.*

Q. 31 A particle is projected in air at an angle β to a surface which itself is inclined at an angle α to the horizontal (figure).



(a) Find an expression of range on the plane surface (distance on the plane from the point of projection at which particle will hit the surface).

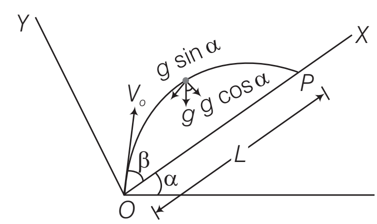
(b) Time of flight.

(c) β at which range will be maximum.

Thinking Process

To solve problems involving projectile motion on an inclined plane, we have to choose two mutually perpendicular axes, one along inclined plane and other perpendicular to the inclined plane.

Ans. Consider the adjacent diagram.



Mutually perpendicular x and y-axes are shown in the diagram.

Particle is projected from the point O.

Let time taken in reaching from point O to point P is T.

(b) Considering motion along vertical upward direction perpendicular to OX.

For the journey O to P.

$$y = 0, u_y = v_0 \sin\beta, a_y = -g\cos\alpha, t = T$$

Applying equation,

$$y = u_y t + \frac{1}{2} a_y t^2$$

$$\Rightarrow \quad 0 = v_0 \sin\beta T + \frac{1}{2}(-g\cos\alpha)T^2$$

$$\Rightarrow \quad T\left[v_0 \sin\beta - \frac{g\cos\alpha}{2} T\right] = 0$$

$$\Rightarrow \quad T = 0, T = \frac{2 v_0 \sin\beta}{g\cos\alpha}$$

As $T = 0$, corresponding to point O

Hence, $$T = \text{Time of flight} = \frac{2 v_0 \sin\beta}{g\cos\alpha}$$

(a) Considering motion along OX.

$$x = L, u_x = v_0 \cos\beta, a_x = -g\sin\alpha$$

$$t = T = \frac{2 v_0 \sin\beta}{g\cos\alpha}$$

$$x = u_x t + \frac{1}{2} a_x t^2$$

$$\Rightarrow \qquad L = v_0 \cos\beta\, T + \frac{1}{2}(-g \sin\alpha) T^2$$

$$\Rightarrow \qquad L = v_0 \cos \beta T - \frac{1}{2} g \sin\alpha\, T^2$$

$$= T\,[v_0 \cos\beta - \frac{1}{2} g \sin\alpha\, T]$$

$$= T\left[v_0 \cos\beta - \frac{1}{2} g \sin \alpha \times \frac{2 v_0 \sin\beta}{g \cos \alpha}\right]$$

$$= \frac{2\, v_0 \sin\beta}{g \cos\alpha}\left[v_0 \cos\beta - \frac{v_0 \sin\alpha \sin\beta}{\cos\alpha}\right]$$

$$= \frac{2\, v_0^2 \sin\beta}{g \cos^2 \alpha}[\cos \beta . \cos\alpha - \sin \alpha . \sin\beta]$$

$$\Rightarrow \qquad L = \frac{2\, v_0^2 \sin\beta}{g \cos^2\, \alpha} \cos (\alpha + \beta)$$

(c) For range (L) to be maximum,

$\sin \beta . \cos (\alpha + \beta)$ should be maximum.

Let, $\qquad Z = \sin \beta . \cos (\alpha + \beta)$

$$= \sin \beta\, [\cos \alpha . \cos \beta - \sin \alpha . \sin \beta)$$

$$= \frac{1}{2}[\cos \alpha . \sin 2 \beta - 2 \sin \alpha . \sin^2 \beta]$$

$$= \frac{1}{2}[\sin 2 \beta . \cos \alpha - \sin \alpha\,(1 - \cos 2 \beta)]$$

$$\Rightarrow \qquad z = \frac{1}{2}[\sin 2 \beta . \cos \alpha - \sin \alpha + \sin \alpha . \cos 2 \beta]$$

$$= \frac{1}{2}[\sin 2 \beta . \cos \alpha + \cos 2 \beta . \sin \alpha - \sin \alpha]$$

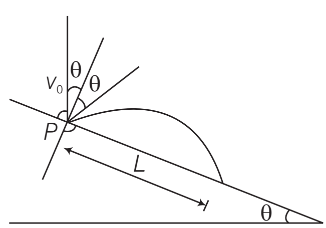
$$= \frac{1}{2}[\sin (2 \beta + \alpha) - \sin \alpha]$$

For z to be maximum,

$$\sin (2 = \beta + \alpha) = \text{maximum} = 1$$

$$\Rightarrow \qquad 2 \beta + \alpha = \frac{\pi}{2} \text{ or, } \beta = \frac{\pi}{4} - \frac{\alpha}{2}$$

Q. 32 A particle falling vertically from a height hits a plane surface inclined to horizontal at an angle θ with speed v_0 and rebounds elastically. Find the distance along the plane where it will hit second time.

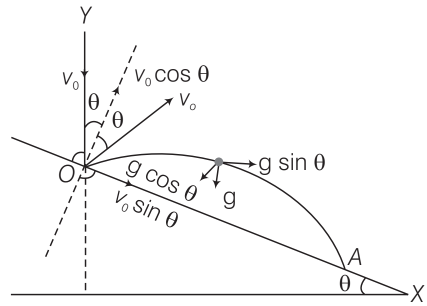


Thinking Process

When particle rebounded elastically speed will remain same.

Ans. Considering x and y-axes as shown in the diagram.

For the motion of the projectile from O to A.



$$y = 0, u_y = v_0 \cos\theta$$
$$a_y = -g \cos\theta, t = T$$

Applying equation of kinematics,

$$y = u_y t + \frac{1}{2} a_y t^2$$

$$\Rightarrow \quad 0 = v_0 \cos\theta\, T + \frac{1}{2}(-g\cos\theta) T^2$$

$$\Rightarrow \quad T, \left[v_0 \cos\theta - \frac{g\cos\theta\, T}{2} \right] = 0$$

$$T = \frac{2 v_0 \cos\theta}{g \cos\theta}$$

As $T = 0$, corresponds to point O

Hence, $\quad T = \dfrac{2v_0}{g}$

Now considering motion along OX.

$$x = L, u_x = v_0 \sin\theta, a_x = g\sin\theta, t = T = \frac{2 v_0}{g}$$

Applying equation of kinematics,

$$x = u_x t + \frac{1}{2} a_x t^2$$

$$\Rightarrow \quad L = v_0 \sin\theta\, t + \frac{1}{2} g \sin\theta\, t^2 = (v_0 \sin\theta)(T) + \frac{1}{2} g \sin\theta\, T^2$$

$$= (v_0 \sin\theta)\left(\frac{2v_0}{g}\right) + \frac{1}{2} g \sin\theta \times \left(\frac{2 v_0}{g}\right)^2$$

$$= \frac{2 v_0^2}{g}\sin\theta + \frac{1}{2} g \sin\theta \times \frac{4v_0^2}{g^2} = \frac{2 v_0^2}{g}[\sin\theta + \sin\theta]$$

$$\Rightarrow \quad L = \frac{4v_0^2}{g}\sin\theta$$

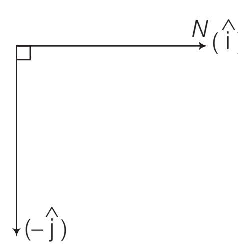
Q. 33 A girl riding a bicycle with a speed of 5 m/s towards north direction, observes rain falling vertically down. If she increases her speed to 10 m/s, rain appears to meet her at 45° to the vertical. What is the speed of the rain? In what direction does rain fall as observed by a ground based observer?

Thinking Process

Draw the vector diagram is for the information given and find a and b. We may draw all vectors in the reference frame of ground based observer.

Ans. Assume north to be $\hat{i}$ direction and vertically downward to be $-\hat{j}$.

Let the rain velocity v_r be $a\,\hat{i} + b\,\hat{j}$.



$$\mathbf{v_r} = a\,\hat{i} + b\,\hat{j}$$

Case I Given velocity of girl $= \mathbf{v_g} = (5\,\text{m/s})\,\hat{i}$

Let $\mathbf{v_{rg}}$ = Velocity of rain w.r.t girl

$$= \mathbf{v_r} - \mathbf{v_g} = (a\hat{i} + b\hat{j}) - 5\hat{i}$$
$$= (a - 5)\,\hat{i} + b\hat{j}$$

According to question rain, appears to fall vertically downward.

Hence, $a - 5 = 0 \Rightarrow a = 5$

***Case* II** Given velocity of the girl, $\mathbf{v}_g = (10\,\text{m/s})\,\hat{\mathbf{i}}$

$\therefore$ $\mathbf{v_{rg}} = \mathbf{v_r} - \mathbf{v_g}$

$= (a\hat{\mathbf{i}} + b\hat{\mathbf{j}}) - 10\hat{\mathbf{i}} = (a - 10)\,\hat{\mathbf{i}} + b\hat{\mathbf{j}}$

According to question rain appears to fall at 45° to the vertical hence $\tan 45° = \dfrac{b}{a - 10} = 1$

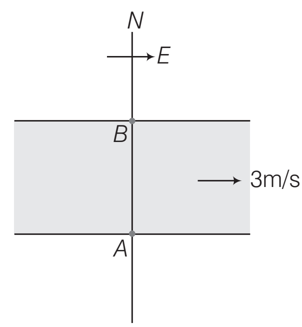
$\Rightarrow$ $b = a - 10 = 5 - 10 = -5$

Hence, velocity of rain $= a\hat{\mathbf{i}} + b\hat{\mathbf{j}}$

$\Rightarrow$ $\mathbf{v}_r = 5\hat{\mathbf{i}} - 5\hat{\mathbf{j}}$

Speed of rain $= |\mathbf{v_r}| = \sqrt{(5)^2 + (-5)^2} = \sqrt{50} = 5\sqrt{2}\,\text{m/s}$

Q. 34 A river is flowing due east with a speed 3 m/s. A swimmer can swim in still water at a speed of 4 m/s (figure).



(a) If swimmer starts swimming due north, what will be his resultant velocity (magnitude and direction)?

(b) If he wants to start from point A on south bank and reach opposite point B on north bank,

(i) which direction should he swim?

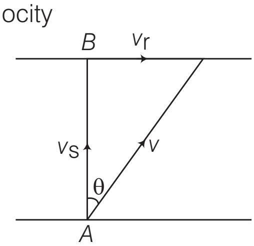
(ii) what will be his resultant speed?

(c) From two different cases as mentioned in (a) and (b) above, in which case will he reach opposite bank in shorter time?

Ans. Given, Speed of the river $(v_r) = 3\,\text{m/s}$ (east)

Speed of swimmer $(v_s) = 4\,\text{m/s}$ (east)

(a) When swimmer starts swimming due north then his resultant velocity

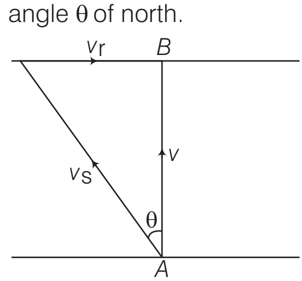


$$v = \sqrt{v_r^2 + v_s^2} = \sqrt{(3)^2 + (4)^2}$$
$$= \sqrt{9 + 16} = \sqrt{25} = 5\,\text{m/s}$$
$$\tan\theta = \frac{v_r}{v_s} = \frac{3}{4}$$
$$= 0.75 = \tan 36°54'$$

Hence, $\theta = 36°54'$ N

(b) To reach opposite points B, the swimmer should swim at an angle θ of north.

Resultant speed of the swimmer



$$v = \sqrt{v_s^2 - v_r^2} = \sqrt{(4)^2 - (3)^2}$$
$$= \sqrt{16 - 9} = \sqrt{7}\ \text{m/s}$$
$$\tan\theta = \frac{v_r}{v} = \frac{3}{\sqrt{7}}$$

$\Rightarrow$ $\theta = \tan^{-1}\left(\dfrac{3}{\sqrt{7}}\right)$ of north

(c) In case (a),

Time taken by the swimmer to cross the river, $t_1 = \frac{d}{v_s} = \frac{d}{4}$ s

In case (b),

Time taken by the swimmer to cross the river

$$t_1 = \frac{d}{v} = \frac{d}{\sqrt{7}}$$

As $\frac{d}{4} < \frac{d}{\sqrt{7}}$, therefore $t_1 < t_2$

Hence, the swimmer will cross the river in shorter time in case (a).

Q. 35 A cricket fielder can throw the cricket ball with a speed v_0. If he throws the ball while running with speed u at an angle θ to the horizontal, find

(a) the effective angle to the horizontal at which the ball is projected in air as seen by a spectator.

(b) what will be time of flight?

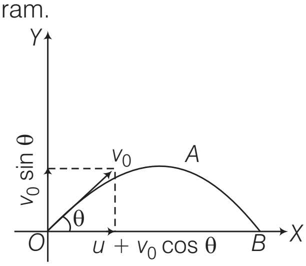
(c) what is the distance (horizontal range) from the point of projection at which the ball will land?

(d) find θ at which he should throw the ball that would maximise the horizontal range as found in (iii).

(e) how does θ for maximum range change if $u > u_0$, $u = u_0$, $u < v_0$?

(f) how does θ in (v) compare with that for $u = 0$ (*i.e.*, 45°)?

Ans. Consider the adjacent diagram.



(a) Initial velocity in

x-direction, $u_x = u + v_0 \cos\theta$

u_y = Initial velocity in y-direction

$= v_0 \sin\theta$

where angle of projection is θ.

Now, we can write

$$\tan\theta = \frac{u_y}{u_x} = \frac{u_0 \sin\theta}{u + u_0 \cos\theta}$$

$$\Rightarrow \qquad \theta = \tan^{-1}\left(\frac{v_0 \sin\theta}{u + v_0 \cos\theta}\right)$$

(b) Let T be the time of flight.

As net displacement is zero over time period T.

$$y = 0, u_y = v_0 \sin\theta, a_y = -g, t = T$$

We know that $$y = u_y t + \frac{1}{2} a_y t^2$$

$\Rightarrow$ $$0 = v_0 \sin\theta\, T + \frac{1}{2}(-g) T^2$$

$\Rightarrow$ $$T\left[v_0 \sin\theta - \frac{g}{2} T\right] = 0 \Rightarrow T = 0, \frac{2v_0 \sin\theta}{g}$$

$T = 0_1$, corresponds to point O.

Hence, $$T = \frac{2u_0 \sin\theta}{g}$$

(c) Horizontal range, $R = (u + v_0 \cos\theta) T = (u + v_0 \cos\theta) \dfrac{2v_0 \sin\theta}{g}$

$$= \frac{v_0}{g}[2u \sin\theta + v_0 \sin 2\theta]$$

(d) For horizontal range to be maximum, $\dfrac{dR}{d\theta} = 0$

$\Rightarrow$ $$\frac{v_0}{g}[2u\cos\theta + v_0 \cos 2\theta \times 2] = 0$$

$\Rightarrow$ $$2u\cos\theta + 2v_0\,[2\cos^2\theta - 1] = 0$$

$\Rightarrow$ $$4v_0 \cos^2\theta + 2u\cos\theta - 2v_0 = 0$$

$\Rightarrow$ $$2v_0 \cos^2\theta + u\cos\theta - v_0 = 0$$

$\Rightarrow$ $$\cos\theta = \frac{-u \pm \sqrt{u^2 + 8v_0^2}}{4v_0}$$

$\Rightarrow$ $$\theta_{max} = \cos^{-1}\left[\frac{-u \pm \sqrt{u^2 + 8v_0^2}}{4v_0}\right]$$

$$= \cos^{-1}\left[\frac{-u + \sqrt{u^2 + 8v_0^2}}{4v_0}\right]$$

(e) If $u = v_0$,

$$\cos\theta = \frac{-v_0 \pm \sqrt{v_0^2 + 8v_0^2}}{4v_0} = \frac{-1 + 3}{4} = \frac{1}{2}$$

$\Rightarrow$ $$\theta = 60°$$

If $u << v_0$, then $8v_0^2 + u^2 \approx 8v_0^2$

$$\theta_{max} = \cos^{-1}\left[\frac{-u \pm 2\sqrt{2}v_0}{4v_0}\right] = \cos^{-1}\left[\frac{1}{\sqrt{2}} - \frac{u}{4v_0}\right]$$

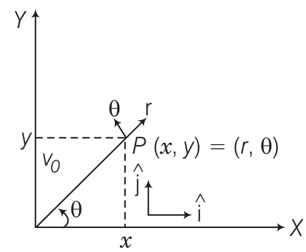
If $u << v_0$, then $$\theta_{max} = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$$

If $u > u_0$ and $u >> v_0$

$$\theta_{max} = \cos^{-1}\left[\frac{-u \pm u}{4v_0}\right] = 0 \Rightarrow \theta_{max} = \frac{\pi}{2}$$

(f) If $u = 0$, $\theta_{max} = \cos^{-1}\left[\dfrac{0 \pm \sqrt{8v^2{}_0}}{4v_0}\right] = \cos^{-1}\left(\dfrac{1}{\sqrt{2}}\right) = 45°$

Q. 36 Motion in two dimensions, in a plane can be studied by expressing position, velocity and acceleration as vectors in cartesian coordinates $\mathbf{A} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}$, where $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ are unit vector along x and y-directions, respectively and A_x and A_y are corresponding components of A. Motion can also be studied by expressing vectors in circular polar coordinates as $\mathbf{A} = A_r\hat{\mathbf{r}} + A_\theta\hat{\theta}$, where $\hat{\mathbf{r}} = \frac{\mathbf{r}}{r} = \cos\theta\,\hat{\mathbf{i}} + \sin\theta\,\hat{\mathbf{j}}$ and $\hat{\theta} = -\sin\theta\,\hat{\mathbf{i}} + \cos\theta\hat{\mathbf{j}}$ are unit vectors along direction in which r and θ are increasing.



(a) Express $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ in terms of $\hat{\mathbf{r}}$ and $\hat{\theta}$.

(b) Show that both $\hat{\mathbf{r}}$ and $\hat{\theta}$ are unit vectors and are perpendicular to each other.

(c) Show that $\frac{d}{dt}(\hat{\mathbf{r}}) = \omega\,\hat{\theta}$, where $\omega = \frac{d\theta}{dt}$ and $\frac{d}{dt}(\hat{\theta}) = -\theta\hat{\mathbf{r}}$.

(d) For a particle moving along a spiral given by $r = a\theta\hat{r}$, where $a = 1$ (unit), find dimensions of a.

(e) Find velocity and acceleration in polar vector representation for particle moving along spiral described in (d) above.

Ans. (a) Given, unit vector $\hat{r} = \cos\theta\hat{i} + \sin\theta\hat{j}$ ...(i)

$\hat{\theta} = -\sin\theta\hat{i} + \cos\theta\hat{j}$ ...(ii)

Multiplying Eq. (i) by $\sin\theta$ and Eq. (ii) with $\cos\theta$ and adding

$$\hat{r}\sin\theta + \hat{\theta}\cos\theta = \sin\theta.\cos\theta\hat{i} + \sin^2\theta\hat{j} + \cos^2\theta\hat{j} - \sin\theta.\cos\theta\hat{i}$$

$$= \hat{j}(\cos^2\theta + \sin^2\theta) = \hat{j}$$

$\Rightarrow$ $\hat{r}\sin\theta + \hat{\theta}\cos\theta = \hat{j}$

By Eq. (i) $\times\cos\theta$ – Eq. (ii) $\times\sin\theta$

$$n(\hat{r}\cos\theta - \hat{\theta}\sin\theta) = \hat{i}$$

(b) $\hat{r}.\hat{\theta} = (\cos\theta\hat{i} + \sin\theta\hat{j}).(-\sin\theta\hat{i} + \cos\theta\hat{j}) = -\cos\theta.\sin\theta + \sin\theta.\cos\theta = 0$

$\Rightarrow$ $\theta = 90°$ Angle between $\hat{\mathbf{r}}$ and $\hat{\theta}$.

(c) Given, $\hat{r} = \cos\theta\hat{i} + \sin\theta\hat{j}$

$$\frac{d\hat{r}}{dt} = \frac{d}{dt}(\cos\theta\hat{i} + \sin\theta\hat{j}) = -\sin\theta.\frac{d\theta}{dt}\hat{i} + \cos\theta.\frac{d\theta}{dt}\hat{j}$$

$$= \omega[-\sin\theta\hat{i} + \cos\theta\hat{j}] \qquad \left[\because \theta = \frac{d\theta}{dt}\right]$$

(d) Given, $\mathbf{r} = a\theta\hat{r}$, here, writing dimensions $[\mathbf{r}] = [a][\theta][\hat{r}]$

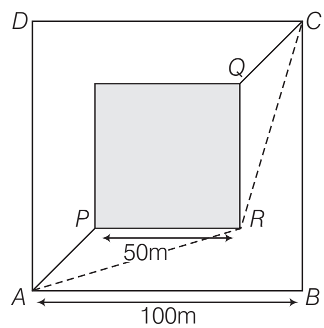
$\Rightarrow$ $L = [a]\,\%\,1 \Rightarrow [a] = L = [M^0L^1T^0]$

(e) Given, $a = 1$ unit $\quad \mathbf{r} = \theta\hat{\mathbf{r}} = \theta\,[\cos\theta\,\hat{\mathbf{i}} + \sin\theta\,\hat{\mathbf{j}}]$

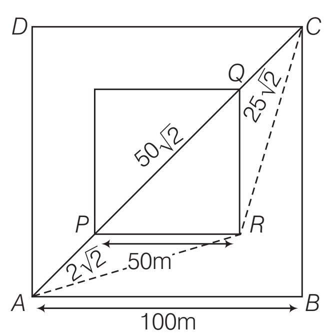
Velocity,
$$v = \frac{d\mathbf{r}}{dt} = \frac{d\theta}{dt}\hat{\mathbf{r}} + \theta\frac{d}{dt}\hat{\mathbf{r}} = \frac{d\theta}{dt}\hat{\mathbf{r}} + \theta\frac{d}{dt}[(\cos\theta\,\hat{\mathbf{i}} + \sin\theta\,\hat{\mathbf{j}})]$$
$$= \frac{d\theta}{dt}\hat{\mathbf{r}} + \theta\left[(-\sin\theta\,\hat{\mathbf{i}} + \cos\theta\,\hat{\mathbf{j}})\frac{d\theta}{dt}\right]$$
$$= \frac{d\theta}{dt}\hat{\mathbf{r}} + \theta\,\hat{\theta}\,\omega = \omega\hat{\mathbf{r}} + \omega\theta\hat{\theta}$$

Acceleration,
$$\mathbf{a} = \frac{d}{dt}[\omega\hat{\mathbf{r}} + \omega\theta\hat{\theta}] = \frac{d}{dt}\left[\frac{d\theta}{dt}\hat{\mathbf{r}} + \frac{d\theta}{dt}(\theta\hat{\theta})\right]$$
$$= \frac{d^2\theta}{dt^2}\hat{\mathbf{r}} + \frac{d\theta}{dt}\cdot\frac{d\hat{\mathbf{r}}}{dt} + \frac{d^2\theta}{dt^2}\theta\hat{\theta} + \frac{d\theta}{dt}\frac{d}{dt}(\theta\hat{\theta})$$
$$= \frac{d^2\theta}{dt^2}\hat{\mathbf{r}} + \omega\,[-\sin\theta\,\hat{\mathbf{i}} + \sin\theta\,\hat{\mathbf{j}}] + \frac{d^2\theta}{dt^2}\theta\hat{\theta} + \frac{\omega d}{dt}(\theta\hat{\theta})$$
$$= \frac{d^2\theta}{dt^2}\hat{\mathbf{r}} + \omega^2\hat{\theta} + \frac{d^2\theta}{dt^2}\times\theta\hat{\theta} + \omega^2\hat{\theta} + \omega^2\theta\,(-\hat{\mathbf{r}})$$
$$\left(\frac{d^2\theta}{dtv^2} - \omega^2\right)\hat{\mathbf{r}} + \left(2\omega^2 + \frac{d^2\theta}{dt^2}\theta\right)\theta$$

Q. 37 A man wants to reach from *A* to the opposite corner of the square *C*. The sides of the square are 100 m. A central square of 50m × 50m is filled with sand. Outside this square, he can walk at a speed 1 m/s. In the central square, he can walk only at a speed of *v* m/s ($v < 1$). What is smallest value of *v* for which he can reach faster *via* a straight path through the sand than any path in the square outside the sand?



Ans. Consider adjacent diagram.



Time taken to go from A to C *via* straight line path $APQC$ through the S and

$$T_{\text{sand}} = \frac{AP + QC}{1} + \frac{PQ}{v} = \frac{25\sqrt{2} + 25\sqrt{2}}{1} + \frac{50\sqrt{2}}{v}$$

$$= 50\sqrt{2} + \frac{50\sqrt{2}}{v} = 50\sqrt{2}\left(\frac{1}{v} + 1\right)$$

Clearly from figure the shortest path outside the sand will be ARC.

Time taken to go from A to C *via* this path

$$T_{\text{outside}} = \frac{AR + RC}{1}\ s$$

Clearly,

$$AR = \sqrt{75^2 + 25^2} = \sqrt{75 \times 75 + 25 \times 25}$$

$$= 5 \times 5\sqrt{9 + 1} = 25\sqrt{10}\ \text{m}$$

$$RC = AR = \sqrt{75^2 + 25^2} = 25\sqrt{10}\ \text{m}$$

$\Rightarrow$

$$T_{\text{outside}} = 2AR = 2 \times 25\sqrt{10}\ \text{s} = 50\sqrt{10}\ \text{s}$$

For

$$T_{\text{sand}} < T_{\text{outside}}$$

$\Rightarrow$

$$50\sqrt{2}\left(\frac{1}{v} + 1\right) < 2 \times 25\sqrt{10}$$

$\Rightarrow$

$$\frac{2\sqrt{2}}{2}\left(\frac{1}{v} + 1\right) < \sqrt{10}$$

$\Rightarrow$

$$\frac{1}{v} + 1 < \frac{2\sqrt{10}}{2\sqrt{2}} = \frac{\sqrt{5}}{2} \times 2 = \sqrt{5}$$

$\Rightarrow$

$$\frac{1}{v} < \frac{\sqrt{5}}{2} \times 2 - 1 \quad \Rightarrow \quad \frac{1}{v} < \sqrt{5} - 1$$

$\Rightarrow$

$$v > \frac{1}{\sqrt{5} - 1} \approx 0.81\ \text{m/s}$$

$\Rightarrow$

$$v > 0.81\ \text{m/s}$$