CHAPTER › 14

Oscillations

KEY NOTES

Periodic and Oscillatory Motion

- In uniform circular motion and orbital motion of planets in the solar system, the motion is repeated after a certain interval of time, hence it is called **periodic motion**.
- If the body is given a small displacement from the equilibrium position, a force comes into play which tries to bring the body back to the equilibrium position, giving rise to **oscillations** or **vibrations**.
- A motion in which a body moves to and fro about a mean position or back and forth about a mean position, termed as **oscillatory motion**.
- Every oscillatory motion is periodic but every periodic motion need not to be oscillatory. e.g. Circular motion is a periodic motion, but it is not oscillatory.
- The description of a periodic motion, in general and oscillatory motion in particular, requires some fundamental concepts, like period, frequency, displacement, amplitude and phase.

Period and Frequency

- The smallest interval of time after which the motion is repeated is called its **period**.
- The reciprocal of period T gives the number of repetitions that occur per unit time. This quantity is called the **frequency** of the periodic motion.
- The relation between frequency ν and period T is given as

$$\nu = 1/T$$

The SI unit of frequency is hertz.

Displacement

- The displacement can be represented by a mathematical function of time.
- In case of periodic motion, this function is periodic in time. One of the simplest periodic function is given by

$$f(t) = A\cos\omega t$$

If the argument of this function ωt is increased by an integral multiple of 2π radians, the value of function remains the same.

The function $f(t)$ is then periodic and its period T is given by

$$T = \frac{2\pi}{\omega}$$

- A linear combination of sine and cosine functions like

$$f(t) = A\sin\omega t + B\cos\omega t \qquad \ldots(i)$$

is also a periodic function with same period T.

Eq. (i) can be written as

$$f(t) = D\sin(\omega t + \phi)$$

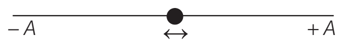
where, $D = \sqrt{A^2 + B^2}$

and $\phi = \tan^{-1}\left(\frac{B}{A}\right)$.

- Therefore, any periodic function can be expressed as a superposition of sine and cosine functions of different time periods with suitable coefficients.

Simple Harmonic Motion

- It is the simplest form of oscillatory motion. This motion arises when the force on the oscillating body is directly proportional to its displacement from the mean position and its direction is always towards the mean position.
- A particle oscillating back and forth about the origin of an X-axis between the limits $+A$ and $-A$ is as shown in figure



This oscillatory motion is said to be **simple harmonic**, if the displacement x of the particle from the origin varies with time as

$$x(t) = A\cos(\omega t + \phi)$$

where, $x(t)$ = displacement x as a function of time t,
A = amplitude,
ω = angular frequency $\left(=\dfrac{2\pi}{T}\right)$,
$(\omega t + \phi)$ = phase (time-dependent)
and ϕ = initial phase constant.

- The **amplitude** A of SHM is the magnitude of maximum displacement of the particle.

Simple Harmonic Motion and Uniform Circular Motion

- Simple harmonic motion can be defined as the projection of uniform circular motion on any diameter of a cycle of reference.
- The particle **velocity and acceleration** during SHM as functions of time are given by

$$v(t) = \frac{d}{dt}x(t) = -\omega A\sin(\omega t + \phi)$$

$$a(t) = \frac{d}{dt}v(t) = -\omega^2 A\cos(\omega t + \phi) = -\omega^2 x(t)$$

- Therefore, we can say that both velocity and acceleration of a body executing simple harmonic motion are periodic functions.
- Velocity amplitude is $v_{\max} = \omega A$ and acceleration amplitude is $a_{\max} = \omega^2 A$.

Force Law and Energy in SHM

- The force acting on a particle of mass m in SHM is given as

$$F(t) = ma = -m\omega^2 x(t)$$

i.e. $F(t) = -kx(t)$

where, $k = m\omega^2$

$\Rightarrow$ $\omega = \sqrt{\dfrac{k}{m}}$

- Like acceleration, force is always directed towards the mean position, hence it is sometimes called the **restoring force in SHM**.
- When a particle oscillating under such a force which is linearly proportional to displacement $x(t)$, then it is called **linear harmonic oscillator**.
- In the real world, the force may contain small additional terms proportional to x^2, x^3, etc. these are called **non-linear oscillators**.
- Both kinetic and potential energies of a particle in SHM vary between zero and their maximum values.
- **Kinetic energy** K of particle performing SHM is given by

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2 A^2\sin^2(\omega t + \phi)$$

$$= \frac{1}{2}kA^2\sin^2(\omega t + \phi)$$

- Kinetic energy in SHM is also a periodic function of time, being zero when the displacement is maximum and maximum when the particle is at the mean position.

  Since, the sign of velocity v is immaterial in K, the period of K is $\dfrac{T}{2}$.
- The **potential energy** of a particle executing simple harmonic motion is

$$U(x) = \frac{1}{2}kx^2 = \frac{1}{2}kA^2\cos^2(\omega t + \phi)$$

- The potential energy of a particle executing simple harmonic motion is also periodic, with period $\dfrac{T}{2}$, being zero at the mean position and maximum at the extreme displacements.
- **Total energy of a particle** in SHM is given as

$$E = U + K = \frac{1}{2}kA^2 = \frac{1}{2}m\omega^2 A^2$$

- The total mechanical energy of a harmonic oscillator is thus independent of time as expected for motion under any conservative force.
- Both kinetic and potential energies peak twice during each period of SHM.
- For $x = 0$ (mean position), the energy is kinetic and at the extremes $x = \pm A$, it is all potential energy. This means, in the course of motion between these limits, kinetic energy increases at the expense of potential energy or *vice-versa*.

Some Systems Executing SHM

- There are no physical examples of absolutely pure simple harmonic motion. In practice, we come across systems that execute simple harmonic motion approximately under certain conditions.

  The simplest observable example of SHM is the small oscillations due to a spring and simple pendulum.

Oscillation Due to a Spring

- When a linear simple harmonic oscillator consisting of a block of mass m attached to a spring is pulled or pushed and released, executes simple harmonic motion, whose period of oscillation is given as

$$T = 2\pi\sqrt{\frac{m}{k}}$$

where, k is spring constant.

- Stiff springs have high value of k, while soft springs have low value of k.

Simple Pendulum

- A simple pendulum in practice, consists of a heavy but small sized metallic bob suspended by a light, inextensible and flexible string.
- The motion of a simple pendulum is simple harmonic whose time period is given by

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Damped Simple Harmonic Motion

- When the motion of simple pendulum, swinging in air, dies out eventually because the air drag and the friction at the support oppose the motion of the pendulum and dissipate its energy gradually, then the pendulum is said to be **executing damped oscillations**.
- The displacement equation of the motion of block (mass m) under the influence of damping force is given as

$$x(t) = Ae^{-bt/2m} \cdot \cos(\omega' t + \phi)$$

where, A is amplitude and ω' is the angular frequency of the damped oscillator is given by $\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$.

- In damped oscillations, the energy of the system is dissipated continuously but for small damping, the oscillations remain approximately periodic.
- The mechanical energy E of the damped oscillator is given by $E(t) = \frac{1}{2}kA^2e^{-bt/m}$
- The mechanical energy in a real oscillating system decreases during oscillation because external force, such as drag inhibit the oscillations and transfer mechanical energy to thermal energy.
- The damping force depends on the nature of the surrounding medium. If the block is immersed in a liquid, the magnitude of damping will be much greater and the dissipation of energy much faster.
- The damping force is generally proportional to velocity of the bob and it acts opposite to the direction of velocity. If the damping force is denoted by $\mathbf{F}_d$, we have,

$$\mathbf{F}_d = -b\mathbf{v}$$

where, the positive constant b depends on characteristics of the medium, size and shape of block.

Note The oscillation whose amplitude does not change with time are called **undamped oscillations**.

Forced Oscillations and Resonance

- When a system (such as a simple pendulum or block attached to a spring) is displaced from its equilibrium position and released, it oscillates with its natural frequency ω and the oscillations are called **free oscillations**.
- All free oscillations eventually die out because of the ever present damping forces. However, an external agency can maintain these oscillations, these are called **forced** or **driven oscillations**.
- When an external periodic force is applied, then displacement equation of forced oscillation is given as $x(t) = A\cos(\omega_d t + \phi)$

where, t is the time measured from the moment when we apply the periodic force.

- The amplitude A is a function of the forced frequency ω_d and the natural frequency ω and given as

$$A = \frac{F_o}{\{m^2(\omega^2 - \omega_d^2)^2 + \omega_d^2 b^2\}^{1/2}}$$

where, F_0 is the amplitude of external periodic force and m is mass of the particle.

- The value of phase difference ϕ is given as

$$\tan\phi = \frac{-v_0}{\omega_d x_0}$$

where, v_0 and x_0 are the velocity and the displacement of the particle at $t = 0$.

- **For small damping, driving frequency far from natural frequency**

In this case, $\omega_d b$ will be much smaller than $m(\omega^2 - \omega_d^2)$, hence amplitude A is given as

$$A = \frac{F_0}{m(\omega^2 - \omega_d^2)}$$

- **When driving frequency is close to natural frequency**

If ω_d is very close to ω, $m(\omega^2 - \omega_d^2)$ would be much less than $\omega_d b$, for any reasonable value of b, then amplitude A becomes

$$A = \frac{F_0}{\omega_d b}$$

- The phenomenon of increase in amplitude, when the driving force is close to the natural frequency of the oscillator is called **resonance**.

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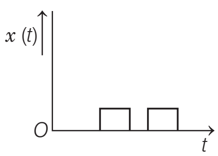
MULTIPLE CHOICE QUESTIONS

TOPIC 1 ~ *Periodic and Oscillatory Motion*

1 The rotation of earth about its axis is
(a) periodic motion
(b) simple harmonic motion
(c) non-periodic motion
(d) None of the above

2 Choose x-t graph for an insect climbing up a ramp uniformly and sliding down then comes back to initial point and repeats the process identically.

(a)


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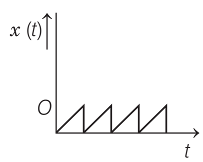

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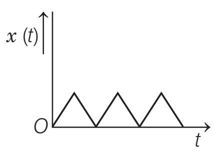

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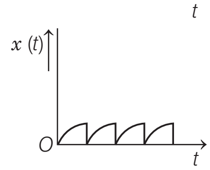


3 On an average, a human heart is found to beat 75 times in a minute. The beat frequency and period of human heart are respectively
(a) 1 Hz, 0.8 s (b) 1 Hz, 1 s
(c) 1.25 Hz, 0.8 s (d) 2 Hz, 0.5 s

4 The periodic function $f(t) = A \sin \omega t$ repeats itself after
(a) 2π (b) 3π (c) π (d) $\pi/2$

5 A particle perform oscillatory motion with amplitude 4 cm. If the time period of particle is 1 s, then time taken by the particle to reach 2 cm from the mean position is given by
(a) $(1/2)$ s (b) $(1/4)$ s
(c) $(1/12)$ s (d) $(1/6)$ s

6 Displacement of a particle in periodic motion is expressed as $x(t) = 20 \cos \omega t$. If the time period of particle motion is 4s, then displacement of the particle in 1 s will be
(a) 10 m (b) 15 m
(c) 0 (d) 20 m

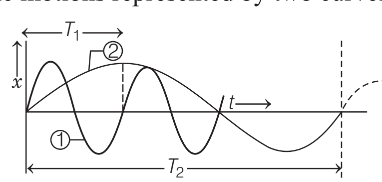
7 The function $\log \omega t$
(a) is a periodic function
(b) is a non-periodic function
(c) could represents oscillatory motion
(d) can represents circular motion

8 Choose the periodic function from the following.
(a) $A \sin^3 (\omega t)^2$ (b) $\sin \omega t + \cos \omega t$
(c) $\tan(\omega t)^3$ (d) $e^{\omega t}$

TOPIC 2~ *Simple Harmonic Motion*

9 Motion of a ball bearing inside a smooth curved bowl, when released from a point slightly above the lower point, is
(a) simple harmonic motion (SHM)
(b) non-periodic motion
(c) parabolic
(d) periodic but not SHM

10 The relation between the time period of two simple harmonic motions represented by two curves is



(a) $T_1 = 4T_2$ (b) $T_2 = 4T_1$ (c) $T_2 = 2T_1$ (d) $T_1 = 2T_2$

11 The distance covered by a particle undergoing SHM in one time period is (amplitude $= A$)

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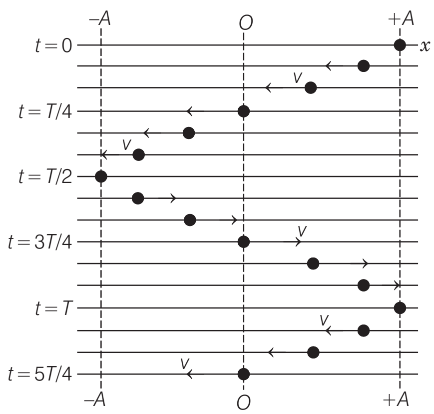
(a) zero (b) A (c) $2A$ (d) $4A$

12 Figure shows snapshots of a particle moving between $+A$ and $-A$ about origin (at $x = 0$) at different instants. The particle moves in a way that velocity is maximum at $x = 0$ and minimum at $x = \pm A$.



The correct displacement equation for the motion of the particle is

(a) $A \sin \omega t$ (b) $A \cos \omega t$
(c) $A \tan \omega t$ (d) $A \cot \omega t$

13 When two displacements represented by $y_1 = a \sin \omega t$ and $y_2 = b \cos \omega t$ are superimposed, the motion is

CBSE AIPMT 2015

(a) not a simple harmonic
(b) simple harmonic with amplitude $\frac{a}{b}$
(c) simple harmonic with amplitude $\sqrt{a^2 + b^2}$
(d) simple harmonic with amplitude $\frac{(a+b)}{2}$

14 The displacement of a particle executing simple harmonic motion is given by
$y = A_0 + A \sin \omega t + B \cos \omega t$.
Then, the amplitude of its oscillation is given by

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(a) $\sqrt{A^2 + B^2}$ (b) $\sqrt{A_0^2 + (A+B)^2}$
(c) $A + B$ (d) $A_0 + \sqrt{A^2 + B^2}$

15 Two simple harmonic motions are represented by

$$y_1 = 5\,(\sin 2\pi t + \sqrt{3} \cos 2\pi t)$$

and $$y_2 = 5 \sin\left(2\pi t + \frac{\pi}{4}\right).$$

The ratio of their amplitudes is

(a) 1 : 1 (b) 2 : 1 (c) 1 : 3 (d) $\sqrt{3} : 1$

16 Which of the following is incorrect when function $x = A \sin^2 \omega t + B \cos^2 \omega t + C \sin \omega t \cos \omega t$ represents SHM?

(a) if $A = 0, B = 0$, it will be in SHM
(b) if $A = -B, C = 2B$, amplitude $= |B\sqrt{2}|$
(c) if $A = B, C = 0$, amplitude $= |B|$
(d) if $A = B, C = 2B$, it will be in SHM

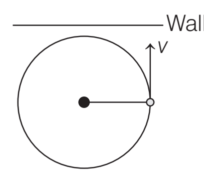
17 A body executing simple harmonic motion has a periodic time of 3 s. After how much time from $t = 0$, its displacement will be half of its amplitude?

(a) $(1/8)$ s (b) $(1/6)$ s (c) $(1/4)$ s (d) $(1/3)$ s

18 Two particles execute SHM of the same amplitude and frequency along the same straight line. If they pass one another when going in opposite directions, each time their displacement is half their amplitude, the phase difference between them is

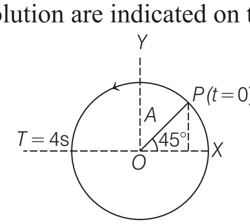
(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{2\pi}{3}$

19 A ball is moving in uniform circular motion in a horizontal plane, the shadow of ball on the wall will execute



(a) projectile motion
(b) uniform circular motion
(c) simple harmonic motion
(d) non-uniform circular motion

20 Figure depicts a circular motion. The radius of the circle, period of revolution, initial position and the sense of revolution are indicated on the figure.



The simple harmonic motion of the X-projection of the radius vector of the rotating particle P is

(a) $A \cos\left(\frac{2\pi}{4} t + \frac{\pi}{4}\right)$ (b) $A \cos\left(\frac{\pi}{4} t + \frac{\pi}{4}\right)$
(c) $A \cos\left(\frac{\pi}{3} t + \frac{\pi}{2}\right)$ (d) None of these

21 The equation of a simple harmonic motion is given by $y = 3 \sin(50t - x)$, where x and y are in metres and t is in seconds, the maximum particle velocity in ms^{-1} is

(a) 3 (b) 50 (c) 150 (d) 25

22 The piston in the cylinder head of a locomotive has a stroke (twice the amplitude) of 2.0 m. If the piston moves with simple harmonic motion with an angular frequency of 100 rad min^{-1}, what is its maximum speed?

(a) 50 m min^{-1} (b) 100 m min^{-1}
(c) 200 m min^{-1} (d) 75 m min^{-1}

23 Maximum acceleration in SHM is ***JIPMER 2019***

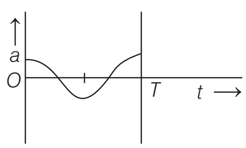
(a) $\omega^2 A$ (b) $\dfrac{\omega^2 A}{2}$
(c) $\omega^2 A^2$ (d) 0

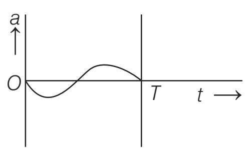
24 Two simple harmonic motions of angular frequency 100 rads^{-1} and 1000 rads^{-1} have the same displacement amplitude. The ratio of their maximum accelerations is

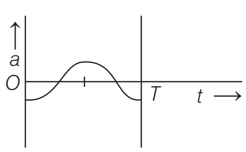
(a) 1 : 10 (b) $1:10^2$
(c) $1:10^3$ (d) $1:10^4$

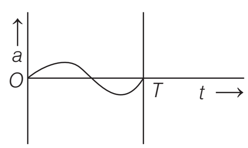
25 The oscillation of a body on a smooth horizontal surface is represented by the equation $x = A \cos \omega t$, where x = displacement at time t and ω = frequency of oscillation.

Which one of the following graphs shows correctly the variation of a with t? ***CBSE AIPMT 2014***

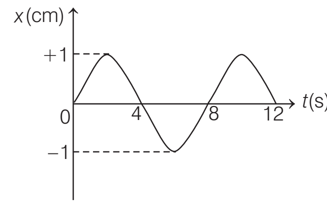
(a) 

(b) 

(c) 

(d) 

26 The x-t graph of a particle undergoing simple harmonic motion is shown below.



The acceleration of the particle at $t = \dfrac{4}{3}$ s is

(a) $\dfrac{\sqrt{3}}{32}\pi^2 \text{ cms}^{-2}$ (b) $-\dfrac{\pi^2}{32}\text{cms}^{-2}$
(c) $\dfrac{\pi^2}{32}\text{cms}^{-2}$ (d) $-\dfrac{\sqrt{3}}{32}\pi^2\text{cms}^{-2}$

27 A pendulum is hung from the roof of a sufficiently high building and is moving freely to and fro like a simple harmonic oscillator. The acceleration of the bob of the pendulum is 20 ms^{-2} at a distance of 5 m from the mean position. The time period of oscillation is ***NEET 2018***

(a) 2 s (b) π s (c) 2π s (d) 1 s

28 A particle is executing a simple harmonic motion. Its maximum acceleration is α and maximum velocity is β, then its time period of vibration will be ***CBSE AIPMT 2015***

(a) $\dfrac{\beta^2}{\alpha^2}$ (b) $\dfrac{\alpha}{\beta}$ (c) $\dfrac{\beta^2}{\alpha}$ (d) $\dfrac{2\pi\beta}{\alpha}$

29 A particle executes linear simple harmonic motion with an amplitude of 3 cm. When the particle is at 2 cm from the mean position, the magnitude of its velocity is equal to that of its acceleration. Then, its time period in seconds is ***NEET 2017***

(a) $\dfrac{\sqrt{5}}{\pi}$ (b) $\dfrac{\sqrt{5}}{2\pi}$ (c) $\dfrac{4\pi}{\sqrt{5}}$ (d) $\dfrac{2\pi}{\sqrt{3}}$

TOPIC 3 ~ *Force Law and Energy in SHM*

30 In simple harmonic motion, the force

(a) is constant in magnitude only
(b) is constant in direction only
(c) varies in magnitude as well as in direction
(d) is constant in both magnitude and direction

31 For simple harmonic motion of an object of mass m,

(a) $\mathbf{F} = -m\omega^2 x$
(b) $\mathbf{F} = -m\omega x$
(c) force always acts in the opposite direction of displacement
(d) Both (a) and (c)

32 In SHM,

(a) PE is stored due to elasticity of system
(b) KE is stored due to inertia of system
(c) Both KE and PE are stored by virtue of elasticity of system.
(d) Both (a) and (b)

33 A mass of 1 kg is executing SHM which is given by $x = 6.0 \cos\left(100t + \frac{\pi}{4}\right)$ cm. What is the maximum kinetic energy? ***JIPMER 2018***

(a) 3 J (b) 6 J (c) 9 J (d) 18 J

34 The expression for displacement of an object in SHM is $x = A\cos\omega t$. The potential energy at $t = T/4$ is

(a) $\frac{1}{2}kA^2$ (b) $\frac{1}{8}kA^2$

(c) $\frac{1}{4}kA^2$ (d) zero

35 In simple harmonic motion, let the time period of variation of potential energy is T_1 and time period of variation of position is T_2, then relation between T_1 and T_2 is ***JEE Main 2017***

(a) $T_1 = T_2$ (b) $T_1 = 2T_2$

(c) $2T_1 = T_2$ (d) None of these

36 The total energy of a particle executing simple harmonic motion depends on its

I. amplitude.

II. period.

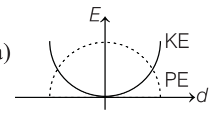
III. displacement.

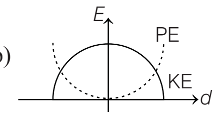
Choose the correct option from the following options.

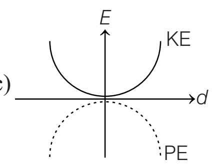
(a) Both I and II (b) Both II and III

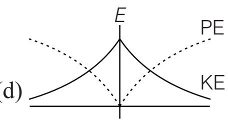
(c) Both I and III (d) I, II and III

37 For a particle executing SHM, a graph is plotted between its kinetic energy and potential energy against its displacement d. Which one of the following represents these correctly? (graphs are schematic and not drawn to scale) ***JEE Main 2015***

(a) 

(b) 

(c) 

(d) 

38 A particle free to move along the X-axis has potential energy given as

$$U(x) = k[1 - \exp(-x^2)] \qquad (\text{for } -\infty < x < +\infty)$$

where, k is a positive constant of appropriate dimensions. Then,

(a) at points away from origin, the particle is in equilibrium

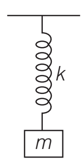
(b) for any finite non-zero value of x, there is a force directed away from the origin

(c) its total mechanical energy is $k/2$ and it is equal to its kinetic energy at origin

(d) the motion of the particle is simple harmonic

TOPIC 4 ~ *Some Systems Executing SHM*

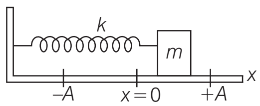
39 A block is left in the equilibrium position as shown in the figure. If now it is stretched by $\frac{mg}{k}$, the net stretch of the spring is



(a) $\frac{mg}{k}$ (b) $\frac{mg}{2k}$

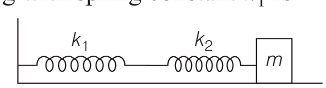
(c) $\frac{2mg}{k}$ (d) $\frac{mg}{4k}$

40 A block is in SHM on a frictionless surface as shown in the figure. The position $x = 0$ show the unstretched position of the spring. If the spring is, then stretched and the block is released at $x = A$. Then, the speed of block at $x = 0$ is



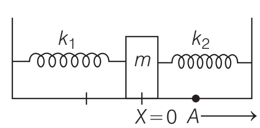
(a) 0 (b) $A^2\sqrt{\frac{k}{m}}$ (c) $A^2\sqrt{\frac{2k}{m}}$ (d) $A\sqrt{\frac{k}{m}}$

41 The mass m as shown in the figure oscillates in simple harmonic motion with amplitude A. The extension in the spring with spring constant k_1 is



(a) $\frac{k_1 A}{k_2}$ (b) $\frac{k_2 A}{k_2}$ (c) $\frac{k_1 A}{k_1 + k_2}$ (d) $\frac{k_2 A}{k_1 + k_2}$

42 In the figure shown, the block is moved side ways by a distance A. The magnitude of net force on the block is



(a) $(k_1 - k_2)A$ (b) $(k_2 - k_1)A$

(c) $(k_1 + k_2)A$ (d) $\left(\frac{k_1 + k_2}{2}\right)A$

43 A system containing a ball is oscillating on a frictionless horizontal plane. The position of the mass when its potential energy and its kinetic energy both are equal, is (let A is the amplitude of oscillation)

(a) A (b) $A/\sqrt{2}$

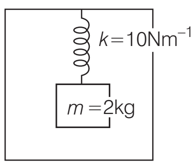
(c) $A/2$ (d) $A/\sqrt{3}$

44 A block attached to a spring is executing SHM. Let the time period of variation of velocity is T_1 and time period of variation of kinetic energy is T_2. The relation between T_1 and T_2 is

(a) $T_1 = T_2$ (b) $T_1 = 2T_2$

(c) $T_1 = T_2/2$ (d) None of these

45 A spring mass system is hanging from a ceiling of an elevator in equilibrium as shown. The elevator suddenly starts accelerating with acceleration 9 ms $^{-2}$, then the frequency of oscillation is



(a) 10 Hz (b) 4 Hz (c) 2.8 Hz (d) 0 Hz

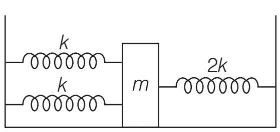
46 Two identical blocks A and B, each of mass m resting on smooth floor, are connected by a light spring of natural length L and the spring constant k, with the spring at its natural length. A third identical block at C (mass m) moving with a speed v along the line joining A and B collides with A. (Consider the collision to be elastic in nature)

The maximum compression in the spring is equal to

(a) $v\sqrt{\dfrac{m}{2k}}$ (b) $m\sqrt{\dfrac{v}{2k}}$

(c) $\sqrt{\dfrac{mv}{k}}$ (d) $\dfrac{mv}{2k}$

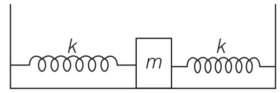
47 The time period of system shown below is

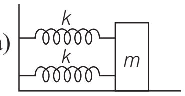


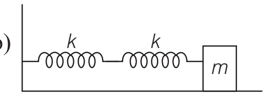
(a) $2\pi\sqrt{\dfrac{m}{3k}}$ (b) $2\pi\sqrt{\dfrac{2m}{k}}$

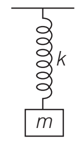
(c) $2\pi\sqrt{\dfrac{m}{4k}}$ (d) $2\pi\sqrt{\dfrac{m}{k}}$

48 The time period of the given spring mass system is equal to that of system



(a) 

(b) 

(c) 

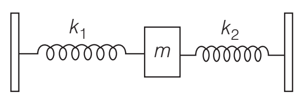
(d) None of these

49 A body mass m is attached to the lower end of a spring whose upper end is fixed. The spring has negligible mass. When the mass m is slightly pulled down and released, it oscillates with a time period of 3s. When the mass m is increased by 1 kg, the time period of oscillations becomes 5s. The value of m in kg is ***NEET 2016***

(a) $\dfrac{3}{4}$ (b) $\dfrac{4}{3}$

(c) $\dfrac{16}{9}$ (d) $\dfrac{9}{16}$

50 Two spring of force constants k_1 and k_2 are connected to a mass m as shown in figure. The frequency of oscillation of the mass is f. If both k_1 and k_2 are made four times their original values, the frequency of oscillation becomes



(a) $f/2$ (b) $f/4$

(c) $4f$ (d) $2f$

51 When a body of mass 1.0 kg is suspended from a certain light spring hanging vertically, its length increases by 5 cm. By suspending 2.0 kg block to the spring and if the block is pulled through 10 cm and released, the maximum velocity in it (in ms^{-1}) is (use, acceleration due to gravity $= 10\,\text{ms}^{-2}$)

(a) 0.5 (b) 1

(c) 2 (d) 4

52 The ratio of frequencies of two pendulums are 2 : 3, then their lengths are in ratio

(a) $\sqrt{2/3}$ (b) $\sqrt{3/2}$

(c) 4/9 (d) 9/4

53 A mass falls from a height h and its time of fall t is recorded in terms of time period T of a simple pendulum. On the surface of earth, it is found that $t = 2T$. The entire set up is taken on the surface of another planet whose mass is half of earth and radius the same. Same experiment is repeated and corresponding times are noted as t' and T'. ***NEET (Odisha) 2019***

(a) $t' = \sqrt{2}\,T'$ (b) $t' > 2\,T'$ (c) $t' < 2T'$ (d) $t' = 2\,T'$

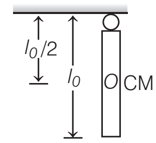
54 A simple pendulum is suspended from the ceiling of a lift. When the lift is at rest, its time period is T. With what acceleration should the lift be accelerated upwards in order to reduce its period to $T/2$? (Take, g is acceleration due to gravity).

(a) $2g$ (b) $3g$
(c) $4g$ (d) g

55 A simple pendulum has a time period T_1 when it is on the earth's surface and T_2 when it is taken to a height $2R$ above the earth's surface, where R is the radius of the earth. The value of T_1/T_2 is ***AIIMS 2018***

(a) 1/9 (b) 1/3
(c) $\sqrt{3}$ (d) 9

56 A uniform rod of mass m and length l_0 is pivoted at one end and is hanging in the vertical direction. The period of small angular oscillations of the rod is ***JIPMER 2018***



(a) $T = 3\pi\sqrt{\frac{2l_0}{3g}}$ (b) $T = 4\pi\sqrt{\frac{l_0}{3g}}$
(c) $T = 4\pi\sqrt{\frac{2l_0}{3g}}$ (d) $T = 2\pi\sqrt{\frac{2l_0}{3g}}$

57 A simple pendulum oscillating in air has period T. The bob of the pendulum is completely immersed in a non-viscous liquid. The density of the liquid is $\frac{1}{16}$th of the material of the bob. If the bob is inside liquid all the time, its period of oscillation in this liquid is ***JEE Main 2019***

(a) $2T\sqrt{\frac{1}{10}}$ (b) $2T\sqrt{\frac{1}{14}}$ (c) $4T\sqrt{\frac{1}{14}}$ (d) $4T\sqrt{\frac{1}{15}}$

TOPIC 5 ~ *Free, Forced and Damped Oscillations; Resonance*

58 The natural frequencies of vibration of a building depend on

(i) its height and other size parameters.
(ii) the nature of building materials.

The correct option is/are

(a) Only (i) (b) Only (ii)
(c) Both (i) and (ii) (d) Neither (i) nor (ii)

59 The value of amplitude of the forced oscillation when damping is small and ω_d is far away from ω, where ω_d = driving frequency, ω = natural frequency and F_0 = amplitude of applied periodic force.

(a) $\frac{F_0}{m\omega^2}$

(b) $\frac{F_0}{m(\omega^2 - \omega_d^2)}$

(c) $\frac{F_0}{\omega_d b}$

(d) $\frac{F_0}{m\omega^2 d}$

60 The value of maximum possible amplitude in the case of forced oscillations when driving frequency is close to natural frequency, is

(a) $\frac{F_0}{m(\omega^2 - \omega_d^2)}$ (b) $\frac{F_0}{\omega_d b}$

(c) $\frac{F_0}{m\omega^2}$ (d) $\frac{-F_0}{m\omega_d^2}$

61 The amplitude of a damped oscillator decreases to 0.9 times its original magnitude is 5 s. In another 10 s, it will decrease to α times its original magnitude, where α equals to ***JEE Main 2013***

(a) 0.7 (b) 0.81 (c) 0.729 (d) 0.6

62 A damped harmonic oscillator has a frequency of 5 oscillations per second. The amplitude drops to half its value for every 10 oscillations. The time it will take to drop to $\frac{1}{1000}$ of the original amplitude is close to ***JEE Main 2019***

(a) 20 s (b) 50 s (c) 100 s (d) 10 s

SPECIAL TYPES QUESTIONS

I. Assertion and Reason

■ **Direction** (Q. Nos. 63-77) *In the following questions, a statement of Assertion is followed by a corresponding statement of Reason. Of the following statements, choose the correct one.*

(a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
(b) Both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
(c) Assertion is correct but Reason is incorrect.
(d) Assertion is incorrect but Reason is correct.

63 **Assertion** Vibrations and oscillations are two different types of motion.

Reason For vibration, frequency is more and for oscillation, the frequency is less.

64 **Assertion** $x(t) = A \sin \omega t$ is periodic in nature but cannot represent an oscillatory motion.

Reason $\sin \theta$ is a sinusoidal periodic function.

65 **Assertion** $x = A \cos \omega t$ and $x = A \sin \omega t$ can represent same motion depending on initial position of particle.

Reason If the argument of $x = A \cos \omega t$, i.e. ωt is increased by 2π radian the value of x remains same.

66 **Assertion** $x = A \cos \omega t$ represents a periodic function. The value of x varies between $+A$ and $-A$.

Reason Amplitude is a vector quantity.

67 **Assertion** For an oscillatory motion, the equilibrium position can be represented by saying that it is a point for periodic motion where net external force on the body is zero.

Reason For an oscillatory motion, if the body is displaced from the equilibrium position, a restoring force will arise that will try to bring back the body to equilibrium position.

68 **Assertion** If the amplitude of SHM of a spring mass system is increased, then time period of SHM will remain constant.

Reason If amplitude is increased, body will have to travel more distance to complete one oscillation.

69 **Assertion** A block of mass m attached a to stiff spring have large oscillation frequency.

Reason Stiff spring have high value of spring constant k.

70 **Assertion** If a pendulum is falling freely, then its time period becomes zero.

Reason Freely falling body has the acceleration equal to g. ***AIIMS 2018***

71 **Assertion** In damped oscillations, the motion is periodic.

Reason In damped oscillations, the amplitude decreases due to dissipative forces.

72 **Assertion** In damped oscillations, the total mechanical energy remain constant.

Reason Total mechanical energy of oscillator executing SHM is given by $\frac{1}{2} kA^2$,where A is amplitude at time t.

73 **Assertion** The motion of a simple pendulum dies out gradually due to air drag and friction at the support.

Reason For small damping also, the oscillations are non-periodic in nature.

74 **Assertion** Free oscillations cannot die out with time.

Reason Swinging of a child in a swing (with an external push) is the example of forced oscillation.

75 **Assertion** In forced oscillation, the external force is constant.

Reason In forced oscillation, external force helps in sustaining the oscillations.

76 **Assertion** In resonance, amplitude is infinity.

Reason At resonance, driving frequency is equal to natural frequency of the system.

77 **Assertion** The army troops are suggested to break their march on a hanging bridge.

Reason Due to resonance, the bridge may collapse.

II. Statement Based Questions

78 I. In sitar and guitar, the strings vibrate and produce sound.
II. Sound wave propagate due to vibration of air molecules.
III. In solids, atoms oscillate to produce the sensation of temperature.
IV. In antennas of TV and satellites transmitters, electrons oscillate to convey information.

Which of the following statement(s) is/are correct?

(a) I, III and IV (b) Both II and III
(c) Both III and IV (d) I, II, III and IV

79 I. Time period of a spring-mass system depends on its amplitude.
II. Time period of a spring-mass system depends on its mass.
III. Time period of a spring-mass system depends on spring constant.

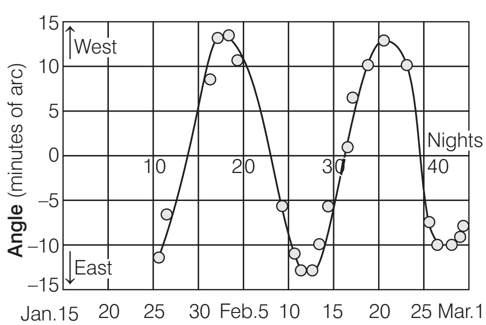
Which of the following statement(s) is/are correct?

(a) Both I and II (b) Both I and III
(c) Both II and III (d) I, II and III

80 Three simple harmonic motions in the same direction having the same amplitude a and same period are superposed. If each differs in phase from the next by 45°, then choose the correct statement.

(a) The resultant amplitude is $(1+\sqrt{2})a$.
(b) The phase of the resultant motion relative to the first is 90°.
(c) The energy associated with the resulting motion is $(3-2\sqrt{2})$ times the energy associated with any single motion.
(d) The resulting motion is not simple harmonic.

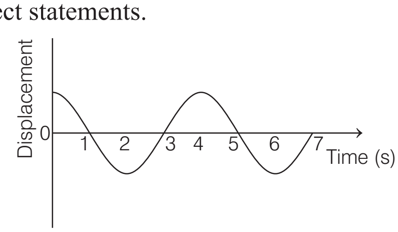
81 In 1610, Galileo found four moons of planet jupiter. He observe the motion of one moon callisto from earth and his data in graph as shown below suggests that its motion is simple harmonic.



Now, choose the correct statement.

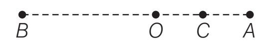
(a) The motion of moon callisto is not simple harmonic in real.
(b) The motion of moon callisto is actually uniform circular motion.
(c) Galileo observed projections of uniform circular motion in a line of plane of motion.
(d) All of the above

82 Displacement *versus* time curve for a particle executing SHM is shown in figure. Choose the correct statements.



(a) Phase of the oscillator is same at $t = 0$ s and $t = 2$ s.
(b) Phase of the oscillator is same at $t = 2$ s and $t = 6$ s.
(c) Phase of the oscillator is same at $t = 1$ s and $t = 7$ s.
(d) Phase of the oscillator is same at $t = 3$ s and $t = 5$ s.

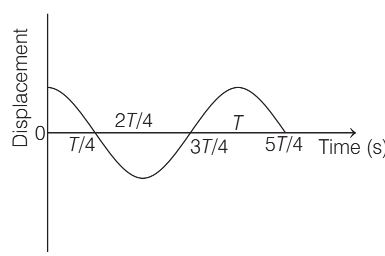
83 A particle is in linear simple harmonic motion between two points A and B, 10 cm apart (figure). Take the direction from A to B as the positive direction and choose the incorrect statements.



$AO = OB = 5$ cm, $BC = 8$ cm

(a) The sign of velocity, acceleration and force on the particle when it is 3 cm away from A going towards B are positive.
(b) The sign of velocity of the particle at C going towards B is negative.
(c) The sign of velocity, acceleration and force on the particle when it is 4 cm away from B going towards A are negative.
(d) The sign of acceleration and force on the particle when it is at point B is negative.

84 The displacement-time graph of a particle executing SHM is shown in figure. Which of the following statement is/are incorrect?



(a) The force is zero at $t = \frac{3T}{4}$.
(b) The acceleration is maximum at $t = \frac{4T}{4}$.
(c) The velocity is maximum at $t = \frac{T}{4}$.
(d) The PE is equal to KE of oscillation at $t = \frac{T}{2}$.

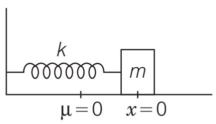
85 A body is performing SHM, then which of the following statement(s) is/are incorrect?

(a) Total energy per cycle is equal to its maximum kinetic energy.
(b) Average kinetic energy per cycle is equal to half of its maximum kinetic energy.
(c) Mean velocity over a complete cycle is equal to $\frac{2}{\pi}$ times of its maximum velocity.
(d) Both (b) and (c)

86 For a SHM, if the maximum potential energy become double, choose the correct statements.
(a) Maximum kinetic energy will become double.
(b) The total mechanical energy will become double.
(c) Both (a) and (b)
(d) Neither (a) nor (b)

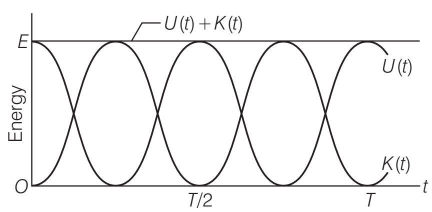
87 A body executes simple harmonic motion. Its Potential Energy (PE), the Kinetic Energy (KE) and Total Energy (TE) were measured as function of displacement x. Then, which of the following statement regarding the body is correct?
(a) KE is maximum, when $x = 0$.
(b) TE is zero, when $x = 0$.
(c) KE is maximum, when x is maximum.
(d) PE is maximum, when $x = 0$.

88 A block is in simple harmonic motion as shown in the figure on a frictionless surface, i.e. $\mu = 0$.



Choose the correct statements.
(a) The kinetic energy varies between a maximum value and zero.
(b) The potential energy varies between a maximum value and zero.
(c) Total energy remains constant.
(d) All of the above

89 The graph below shows the variation of potential energy $U(t)$, kinetic energy $K(t)$ with time t for a particle executing SHM. Choose the correct statement(s).

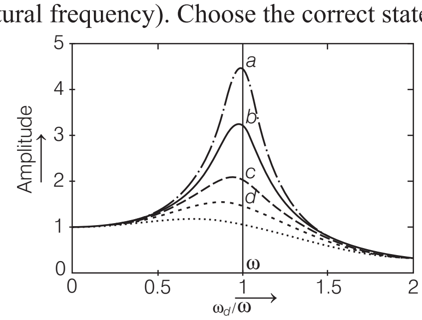


(a) The time periods of variation of potential and kinetic energies are same.
(b) The total mechanical energy is maximum at $t = T/2$.
(c) Kinetic energy is negative and potential energy is positive.
(d) Potential energy is negative and kinetic energy is positive.

90 A vertical spring mass system is taken on moon. It starts oscillating on the moon. Choose the correct statements.
(a) The time period will become $\frac{T_{\text{Earth}}}{\sqrt{6}}$.
(b) The equilibrium position about which spring-mass system oscillates in vertical direction is $\frac{mg}{6k}$ from the unstretched position.
(c) k on moon decreases.
(d) k on moon increases.

91 Choose the correct statements regarding the expression $x(t) = A\,e^{-bt/2m}\cos(\omega' t + \phi)$.
Here, $\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$, $\frac{b}{\sqrt{km}}$ is much less than 1.
(a) $x(t)$ is strictly periodic always.
(b) $x(t)$ is approximately periodic.
(c) Amplitude of damped oscillation represented by above expression is constant.
(d) None of the above

92 Graph below shows variation of amplitude of forced oscillation with respect to frequency of driving force (natural frequency). Choose the correct statement(s).



(a) Amplitude is maximum at $\omega_d = \omega$.
(b) Peak amplitude is maximum for curve a because for curve a damping is minimum.
(c) Both (a) and (b)
(d) None of the above

III. Matching Type

93 Match the Column I (examples of different types of motion) with Column II (type of motion) and select the correct answer from the codes given below.

Column I	Column II
A. Circular motion of a rigid body about a common axis	1. Projectile
B. Motion of a pendulum	2. Rectilinear
C. Motion of a car on a straight road	3. Oscillatory
D. Motion of a ball thrown by a boy at an angle with horizontal	4. Rotational motion

	A	B	C	D		A	B	C	D
(a)	2	3	2	1	(b)	4	3	2	1
(c)	3	4	1	2	(d)	4	3	1	2

94 Match the Column I (quantity) with Column II (value) for an object executing simple harmonic motion in a horizontal plane with displacement given as $x = A\cos\omega t$ and select the correct answer from the codes given below.

Column I	Column II
A. v_{max} / amplitude is equal to	1. $T/8$
B. a_{max} / amplitude is equal to	2. $T/12$
C. If object starts from $x = +A$, then time to reach at $+A\sqrt{2}$	3. ω
D. If object starts from $x = 0$ and move towards right, then the time to reach at $+A/2$	4. ω^2

	A	B	C	D
(a)	3	1	4	2
(b)	3	4	1	2
(c)	4	3	2	1
(d)	3	4	2	1

95 For a forced oscillation, match the Column I (quantity) with Column II (expression) and select the correct answer from the codes given below.

Column I	Column II
A. External periodic force is	1. $a(t) = \frac{-k}{m}x(t) - \frac{b}{m}v(t) + \frac{F_0}{m}\cos\omega_d t$
B. The instantaneous acceleration of the object under forced oscillation is	2. $x(t) = A\cos(\omega_d t + \phi)$
C. The displacement for forced oscillation is	3. $A = \frac{F_0}{\{m^2(\omega^2 - \omega_d^2)^2 + \omega_d^2 b^2\}^{1/2}}$
D. Amplitude for forced oscillation is	4. $F = F_0\cos\omega_d t$

	A	B	C	D		A	B	C	D
(a)	4	1	3	2	(b)	1	2	3	4
(c)	4	1	2	3	(d)	3	2	1	4

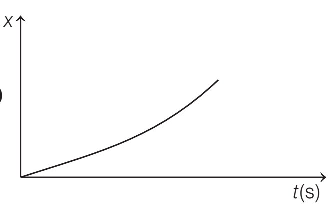
NCERT & NCERT Exemplar

MULTIPLE CHOICE QUESTIONS

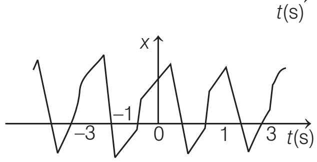
NCERT

96 Figures depict four x-t plots for linear motion of a particle. Which of the plots represent a non-periodic motion?

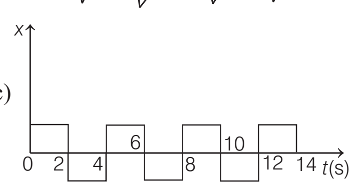
(a)



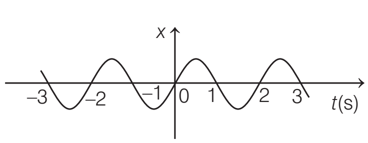
(b)



(c)



(d)



97 The motion of a particle executing simple harmonic motion is described by the displacement function,

$$x(t) = A\cos(\omega t + \phi)$$

If the initial $(t = 0)$ position of the particle is 1 cm and its initial velocity is ω cms^{-1}, what are its amplitude and initial phase angle?

(a) $\sqrt{2}$ cm, $-\frac{\pi}{4}$ (b) 2 cm, $\frac{\pi}{2}$

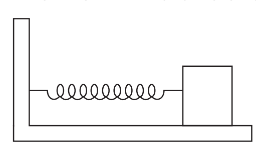
(c) $\sqrt{2}$ cm, $-\frac{\pi}{2}$ (d) 2 cm, $\frac{\pi}{4}$

98 A spring balance has a scale that reads from 0 to 50 kg. The length of the scale is 20 cm. A body suspended from this balance, when displaced and released, oscillates with a period of 0.6 s. What is the weight of the body?

(a) 100 N (b) 200 N

(c) 220 N (d) 250 N

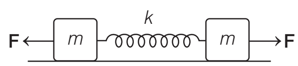
99 A spring having with a spring constant 1200 Nm^{-1} is mounted on a horizontal table as shown in figure.



A mass of 3 kg is attached to the free end of the spring, then the mass is pulled sideways to a distance of 2.0 cm and released. The frequency of oscillation is

(a) 1.6 s^{-1} (b) 3.2 s^{-1} (c) 4.8 s^{-1} (d) 5 s^{-1}

100 Figure shows the same spring with both ends free and attached to a mass m at either end. Each end of the spring in figure is stretched by the same force F.



If masses in figure are released, what is the period of oscillation?

(a) $2\pi\sqrt{\frac{2m}{k}}$ (b) $2\pi\sqrt{\frac{m}{2k}}$ (c) $2\pi\sqrt{\frac{m}{k}}$ (d) $2\pi\sqrt{\frac{2m}{3k}}$

101 The piston in the cylinder head of a locomotive has a stroke (twice the amplitude) of 1.0 m. If the piston moves with simple harmonic motion with an angular frequency of 200 rad min^{-1}, what is its maximum speed?

(a) 25 m min^{-1} (b) 50 m min^{-1}
(c) 100 m min^{-1} (d) 200 m min^{-1}

102 The acceleration due to gravity on the surface of moon is 1.7 ms^{-2}. What is the time period of a simple pendulum on the surface of moon, if its time period on the surface of earth is 3.5 s?
(Take, g on the surface of earth is 9.8 ms^{-2})

(a) 2.4 s (b) 4.2 s (c) 6.2 s (d) 8.4 s

103 You are riding in an automobile of mass 3000 kg. Assuming that you are examining the oscillation characteristics of its suspension system. The suspension sags 15 cm when the entire automobile is placed on it. Also, the amplitude of oscillation decreases by 50% during one complete oscillation. What is the value of the damping constant b for the spring and shock absorber system of one wheel? Assuming that each wheel supports 750 kg.

(a) 1252 kgs^{-1} (b) 1352 kgs^{-1}
(c) 1562 kgs^{-1} (d) 1632 kgs^{-1}

104 A body describes simple harmonic motion with an amplitude of 5 cm and a period of 0.2 s. What is the acceleration and velocity of the body when the displacement is 5 cm?

(a) $-5\pi^2$ ms^{-2}, 0 (b) $2\pi^2$ ms^{-2}, 2ms^{-1}
(c) $1\pi^2$ ms^{-2}, 2ms^{-1} (d) 0, 5 ms^{-1}

NCERT Exemplar

105 The displacement of a particle is represented by the equation $y = 3\cos\left(\frac{\pi}{4} - 2\omega t\right)$. The motion of the particle is

(a) simple harmonic with period $2\pi / \omega$
(b) simple harmonic with period π / ω
(c) periodic but not simple harmonic
(d) non- periodic

106 The displacement of a particle is represented by the equation $y = \sin^3 \omega t$. The motion is

(a) non-periodic
(b) periodic but not simple harmonic
(c) simple harmonic with period $2\pi/\omega$
(d) simple harmonic with period π / ω

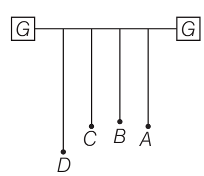
107 The relation between acceleration and displacement of four particles are given below. Which of the particle is executing SHM?

(a) $a_x = +2x$ (b) $a_x = +2x^2$
(c) $a_x = -2x^2$ (d) $a_x = -2x$

108 The displacement of a particle varies with time according to the relation $y = a\sin\omega t + b\cos\omega t$.

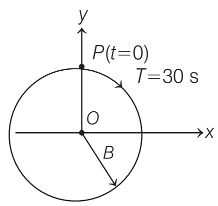
(a) The motion is oscillatory but not SHM
(b) The motion is SHM with amplitude $a + b$
(c) The motion is SHM with amplitude $a^2 + b^2$
(d) The motion is SHM with amplitude $\sqrt{a^2 + b^2}$

109 Four pendulums A, B, C and D are suspended from the same elastic support as shown in figure. A and C are of the same length, while B is smaller than A and D is larger than A. If A is given a transverse displacement, then



(a) D will vibrate with maximum amplitude
(b) C will vibrate with maximum amplitude
(c) B will vibrate with maximum amplitude
(d) All the four will oscillate with equal amplitude

110 Figure shows the circular motion of a particle. The radius of the circle, the period, sense of revolution and the initial position are indicated on the figure. The simple harmonic motion of the x-projection of the radius vector of the rotating particle P is



(a) $x(t) = B\sin\left(\frac{2\pi t}{30}\right)$

(b) $x(t) = B\cos\left(\frac{\pi t}{15}\right)$

(c) $x(t) = B\sin\left(\frac{\pi t}{15} + \frac{\pi}{2}\right)$

(d) $x(t) = B\cos\left(\frac{\pi t}{15} + \frac{\pi}{2}\right)$

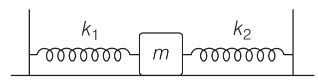
111 The equation of motion of a particle is $x = a\cos(\alpha t)^2$. The motion is

(a) periodic but not oscillatory
(b) periodic and oscillatory
(c) oscillatory but not periodic
(d) Neither periodic nor oscillatory

112 A particle executing SHM has a maximum speed of 30 cms^{-1} and a maximum acceleration of 60 cms^{-2}. The period of oscillation is

(a) π s (b) $\frac{\pi}{2}$ s (c) 2π s (d) $\frac{\pi}{t}$ s

113 When a mass m is connected individually to two springs having spring constants k_1 and k_2, the oscillation frequencies are ν_1 and ν_2. If the same mass is attached to the two springs as shown in figure, the oscillation frequency would be



(a) $\nu_1 + \nu_2$ (b) $\sqrt{\nu_1^2 + \nu_2^2}$

(c) $\left(\frac{1}{\nu_1} + \frac{1}{\nu_2}\right)^{-1}$ (d) $\sqrt{\nu_1^2 - \nu_2^2}$

Answers

> Mastering NCERT with MCQs

1 (a)	2 (c)	3 (c)	4 (a)	5 (c)	6 (c)	7 (b)	8 (b)	9 (a)	10 (b)
11 (d)	12 (b)	13 (c)	14 (a)	15 (b)	16 (c)	17 (c)	18 (d)	19 (c)	20 (a)
21 (c)	22 (b)	23 (a)	24 (b)	25 (c)	26 (d)	27 (b)	28 (d)	29 (c)	30 (c)
31 (d)	32 (d)	33 (d)	34 (d)	35 (c)	36 (a)	37 (b)	38 (d)	39 (c)	40 (d)
41 (d)	42 (c)	43 (b)	44 (b)	45 (c)	46 (a)	47 (c)	48 (a)	49 (d)	50 (d)
51 (b)	52 (d)	53 (d)	54 (b)	55 (b)	56 (d)	57 (d)	58 (c)	59 (b)	60 (b)
61 (c)	62 (a)								

> Special Types Questions

63 (d)	64 (d)	65 (b)	66 (c)	67 (b)	68 (c)	69 (a)	70 (d)	71 (d)	72 (d)
73 (c)	74 (d)	75 (d)	76 (d)	77 (a)	78 (d)	79 (c)	80 (a)	81 (d)	82 (b)
83 (b)	84 (d)	85 (c)	86 (c)	87 (a)	88 (d)	89 (a)	90 (b)	91 (b)	92 (c)
93 (b)	94 (b)	95 (c)							

> NCERT & NCERT Exemplar MCQs

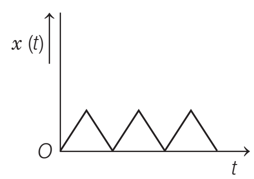
96 (a)	97 (a)	98 (c)	99 (b)	100 (b)	101 (c)	102 (d)	103 (b)	104 (a)	105 (b)
106 (b)	107 (d)	108 (d)	109 (b)	110 (a)	111 (c)	112 (a)	113 (b)		

Hints & Explanations

1 (*a*) The rotation of earth about its axis is periodic because it repeats after a regular interval of time.

However, it is obviously not a to and fro type of motion about a fixed point, hence its motion is not simple harmonic motion.

2 (*c*) The x-t graph for an insect climbing up a ramp and falling down with uniform speed is correctly represented by the graph given in option (c).



As we know that, for uniform motion, the x-t graph is a straight line. So, for the upward motion the graph is represented by straight line with positive slope (considering motion in upward direction as positive) and the downward motion it is represented by a straight line with negative slope.

3 (*c*) The beat frequency of heart $= \dfrac{75}{1\text{ min}} = \dfrac{75}{60}\text{s}^{-1}$

$$= 1.25\text{ Hz}$$

Time period, $T = \dfrac{1}{\text{frequency}} = \dfrac{1}{1.25} = 0.8\text{ s}$

The beat frequency and period of human heart are 1.25 Hz and 0.8 s, respectively.

4 (*a*) A periodic function repeats itself after a time period T.

Given, $f(t) = A\sin\omega t$

If the argument of this function ωt, is increased by an integral multiple of 2π radians. Then, the value of the function remains the same. The given function $f(t)$ is then periodic and repeats itself after every 2π radians.

$$\Rightarrow \quad f(t) = f(t + T)$$

5 (*c*) Given, amplitude, $a = 4$ cm

Time period, $T = 1\text{ s}$,

Displacement, $y = 2\text{ cm}$

Since, particle perform oscillatory motion, hence its displacement equation is

$$y = a\sin\omega t$$

Substituting the given values in the above equation, we get

$$2 = 4\sin\omega t$$

$$\frac{1}{2} = \sin\omega t$$

$$\sin\frac{\pi}{6} = \sin\omega t$$

$$\Rightarrow \quad \omega t = \frac{\pi}{6},\ \frac{2\pi}{T}\cdot t = \frac{\pi}{6} \qquad \left(\because \omega = \frac{2\pi}{T}\right)$$

$$t = \frac{T}{12} = \frac{1}{12}\text{s}$$

6 (*c*) Given, $x(t) = 20\cos\omega t$

$$x(t) = 20\cos\frac{2\pi}{T}\cdot t \qquad \ldots \text{(i)} \left(\because \omega = \frac{2\pi}{T}\right)$$

Given, $\quad T = 4\text{s}$ and $t = 1\text{ s}$

Substituting the given values in Eq. (i), we get

$$\therefore \quad x(t) = 20\cos\frac{2\pi}{4}\cdot 1 = 20\cos\frac{\pi}{2} = 20 \times 0 = 0$$

7 (*b*) The function $\log\omega t$ increases monotonically with time t.Therefore, it cannot repeat its value and is a non-periodic function. It may be noted that as $t \to \infty$, $\log\omega t$ diverges to ∞. Therefore, it cannot represents any kind of physical displacement.

8 (*b*) **In option** (b),

$$(\sin\omega t + \cos\omega t) = \sqrt{2}\left(\frac{1}{\sqrt{2}}\sin\omega t + \frac{1}{\sqrt{2}}\cos\omega t\right)$$

$$= \sqrt{2}\left(\cos\frac{\pi}{4}\sin\omega t + \sin\frac{\pi}{4}\cos\omega t\right)$$

$$= \sqrt{2}\left[\sin\left\{\omega t + \frac{\pi}{4}\right\}\right]$$

[$\because$ using trigonometric identity, $\sin(A + B) = \sin A\cos B + \cos A\sin B$]

Comparing the above equation with the standard equation of periodic function $f(t) = A\sin(\omega t + \phi)$, we can say that, the above function has amplitude $\sqrt{2}$ with phase constant $\pi/4$. Also, it represents a periodic function with period $2\pi/\omega$.

In option (a), $A\sin^3(\omega t)^2$ is not periodic function due to cube of sine value and square of t value.

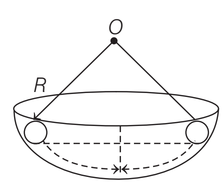
In option (c), $\tan(\omega t)^3$ is not periodic function due to cube of t value.

In option (d), the function $e^{\omega t}$ is not periodic, as it increases with decreasing time.

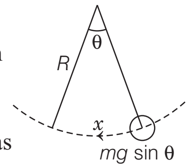
Hence, it never repeats its value.

So, option (b) is correct.

9 (*a*) Consider the motion of the ball inside a smooth curved bowl.



For small angular displacement or slightly released motion, it can be considered as angular SHM. This can be explained as follows



Let the ball is at an angle θ, the restoring force (g sin θ) m acts on it as shown.

$\because \qquad ma = mg\sin\theta \Rightarrow a = g\sin\theta$

$\Rightarrow \qquad \dfrac{d^2x}{dt^2} = -g\sin\theta = -g \times \dfrac{x}{R} \qquad (\because \sin\theta \simeq \theta = x/R)$

$\Rightarrow \qquad d^2x/dt^2 \propto (-x)$

$\therefore \qquad a \propto -x$

Hence, the motion of ball bearing inside the bowl is SHM.

As motion is SHM, hence it must be periodic.

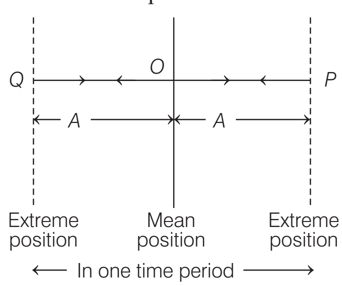
10 (b) From graph, it is clear that time taken to complete one oscillation by SHM represented by curve 1 is equal to time taken to complete one-fourth oscillation by SHM represented by curve 2.

i.e. $\qquad T_1 = \dfrac{T_2}{4} \Rightarrow T_2 = 4T_1$

11 (d) In a simple harmonic motion (SHM), the particle oscillates about its mean position on a straight line.

The particle moves from its mean position O to an extreme position P and then return to its mean position covering same distance of A as shown below.

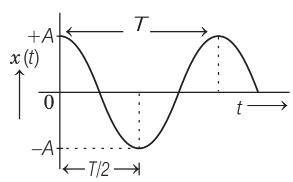
Then by the conservative force, it is moved in opposite direction to a point Q at distance A and then back to mean position covering same distance A. This comprises of one time period as shown below



Hence, in one time period it covers a distance of

$$x = OP + PO + OQ + QO$$
$$= A + A + A + A = 4A$$

12 (b) As the particle is moving between $+A$ and $-A$ with varying speed about origin (at $x = 0$) and by observing snapshots we can draw position-time graph for the given motion.



Graph shows a sinusoidal function x with respect to time t. From figure,at $t = 0$ particle is at $x = +A$ and crosses mean position at $t = T/4$ and reaches other end in negative direction ($-A$) at $t = T/2$.

So, $x(t) = A\cos\omega t$

where, ω is the angular frequency $= \dfrac{2\pi}{T}$.

13 (c) Given, $y_1 = a\sin\omega t$

and $\qquad y_2 = b\cos\omega t = b\sin\left(\omega t + \dfrac{\pi}{2}\right)$

The resultant displacement is given by

$$y = y_1 + y_2 = \sqrt{a^2 + b^2}\sin(\omega t + \phi)$$

Hence, the motion of superimposed wave is simple harmonic with amplitude $\sqrt{a^2 + b^2}$.

14 (a) Standard form of the equation of motion of SHM as a linear combination of sine and cosine functions can be given as $y = a\sin\omega t + b\cos\omega t$ $\qquad$ ...(i)

Let, $\qquad a = d\cos\phi$ and $b = d\sin\phi$

$\Rightarrow \qquad y = d\cos\phi\sin\omega t + d\sin\phi\cos\omega t$

$\qquad = d\sin(\omega t + \phi)$, where $d = \sqrt{a^2 + b^2}$

Here, the displacement of given particle is

$y = A_0 + A\sin\omega t + B\cos\omega t$ $\qquad$ ... (ii)

So, from Eqs. (i) and (ii), we can say that A_0 be the value of mean position for the given particle, at which $y = 0$.

Also, $a = A$ and $b = B$

$\therefore$ Resultant amplitude of the oscillation in given as

$$= \sqrt{A^2 + b^2}$$

15 (b) Given equation,

$$y_1 = 5(\sin 2\pi t + \sqrt{3}\cos 2\pi t)$$
$$= 10\left(\frac{1}{2}\sin 2\pi t + \frac{\sqrt{3}}{2}\cos 2\pi t\right)$$
$$= 10\left(\cos\frac{\pi}{3}\sin 2\pi t + \sin\frac{\pi}{3}\cos 2\pi t\right)$$
$$= 10\left[\sin\left(2\pi t + \frac{\pi}{3}\right)\right] \Rightarrow A_1 = 10$$

Similarly, $y_2 = 5\sin\left(2\pi t + \dfrac{\pi}{4}\right) \Rightarrow A_2 = 5$

Hence, $\dfrac{A_1}{A_2} = \dfrac{10}{5} = \dfrac{2}{1} = 2:1$

So, the ratio of their amplitudes is 2 : 1.

16 (c) Given function

$x = A\sin^2\omega t + B\cos^2\omega t + C\sin\omega t\cos\omega t$

$x = \dfrac{A}{2}(1 - \cos 2\omega t) + \dfrac{B}{2}(1 + \cos 2\omega t) + \dfrac{C}{2}\sin 2\omega t$

For $A = 0, B = 0; x = \dfrac{C}{2}\sin 2\omega t$

It is also represents SHM.

For $A = -B$ and $C = 2B$

$$x = -\frac{B}{2}(1-\cos 2\omega t) + \frac{B}{2}(1+\cos 2\omega t) + \frac{2B}{2}\sin 2\omega t$$

$= B\cos 2\omega t + B\sin 2\omega t$; Amplitude $\sqrt{B^2 + B^2} = |B\sqrt{2}|$

For $A = B$; $C = 0$;

$$x = \frac{A}{2}(1-\cos 2\omega t) + \frac{A}{2}(1+\cos 2\omega t) = A$$

Hence, option (c) is incorrect.

For $A = B, C = 2B$;

$x = B + B\sin 2\omega t$, it also represents SHM.

17 (*c*) We know that, $y = a\sin\omega t$ $\left(\because \omega = \frac{2\pi}{T}\right)$

where, a is the amplitude, $y = a\sin\frac{2\pi}{T}t$

Given, $T = 3\,\text{s}$

So, when the displacement will be half of its amplitude, i.e.

$$y = \frac{a}{2} \Rightarrow \frac{a}{2} = a\sin\frac{2\pi t}{3}$$

$$\frac{1}{2} = \sin\frac{2\pi t}{3}$$

$$\sin\frac{\pi}{6} = \sin\frac{2\pi t}{3}$$

$$\frac{2\pi t}{3} = \frac{\pi}{6}$$

$$t = \frac{1}{4}\,\text{s}$$

18 (*d*) Equation for a particle executing simple harmonic motion is

$$x = A\sin(\omega t + \phi)$$

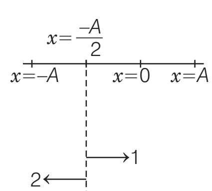
$$A\sin(\omega t + \phi) = \frac{A}{2} \qquad \left[\because \text{Given}, \ x = \frac{A}{2}\right]$$

$$\sin(\omega t + \phi) = \frac{1}{2}$$

$$\sin(\omega t + \phi) = \sin\frac{\pi}{6}$$

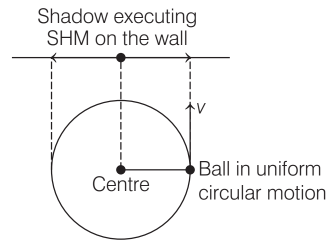
Since, $\delta = \omega t + \phi = \frac{\pi}{6}$ or $\frac{5\pi}{6}$

So, the phase difference of the two particles when they are crossing each other at $x = \frac{A}{2}$ in opposite directions are

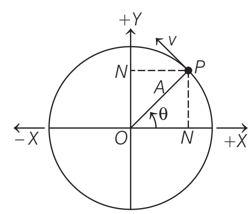


$$\delta = \delta_1 - \delta_2 = \frac{5\pi}{6} - \frac{\pi}{6} = \frac{2\pi}{3}$$

19 (*c*) The ball will appear to execute to and fro motion along a horizontal line with the point of rotation as the mid-point. Thus, the shadow will execute SHM on the wall as shown below.



20 (*a*) At $t = 0$, OP makes an angle of $45° = (\pi/4)$ rad with the (positive direction of) X-axis. After time t, it covers an angle $\frac{2\pi}{T}t$ in the anti-clockwise sense and makes an angle of $\frac{2\pi}{T}t + \frac{\pi}{4}$ with the X-axis.



$\therefore$ The projection of OP on the X-axis at time t is given by

$$x(t) = A\cos\left(\frac{2\pi}{T}t + \frac{\pi}{4}\right)$$

For $T = 4\,\text{s}$,

$$x(t) = A\cos\left(\frac{2\pi}{4}t + \frac{\pi}{4}\right)$$

which is a SHM of amplitude A, period 4s and an initial phase $\frac{\pi}{4}$.

21 (*c*) The equation of a simple harmonic motion is given by

$$y = 3\sin(50t - x) \qquad \ldots\text{(i)}$$

By comparing Eq. (i) with general equation of a simple harmonic motion $y = A\sin(\omega t + \phi)$, we get

Amplitude $A = 3\,\text{m}$

angular frequency, $\omega = 50\,\text{Hz}$

$\therefore$ Maximum particle velocity, $v_{max} = A\omega = 3 \times 50 = 150\,\text{ms}^{-1}$

22 (*b*) Given, angular frequency of the piston,

$$\omega = 100\,\text{rad min}^{-1}$$

and stroke length = 2m

$\therefore$ Amplitude of SHM, $A = \frac{\text{Stroke length}}{2} = \frac{1}{2} = 1\,\text{m}$

Now, $v_{max} = \omega A = 100 \times 1 = 100\,\text{m min}^{-1}$

24 (*b*) Maximum acceleration of object in simple harmonic motion is

$$a_{\max} = \omega^2 A$$

$$\Rightarrow \quad \frac{(a_{\max})_1}{(a_{\max})_2} = \frac{\omega_1^2}{\omega_2^2} \qquad \text{(as } A \text{ remains same)}$$

$$\Rightarrow \quad \frac{(a_{\max})_1}{(a_{\max})_2} = \frac{(100)^2}{(1000)^2} = \left(\frac{1}{10}\right)^2 = 1:10^2$$

So, the ratio of their maximum acceleration is $1 : 10^2$.

25 (*c*) The oscillation of a body on a smooth horizontal surface is represented by the equation, $x(t) = A\cos\omega t$

$$\Rightarrow \quad v(t) = \frac{d}{dt}\{x(t)\} = -\omega A\sin\omega t$$

$$\text{and} \quad a(t) = \frac{d}{dt}\{v(t)\} = -\omega^2 A\cos\omega t \qquad \ldots\text{(i)}$$

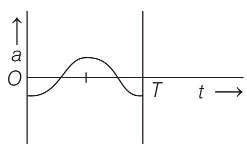
So, for the corresponding *a*-*t* graph for the given body, let us first calculate the value of *a* at different values of *t*, using Eq. (i), we get

If $t = 0$, then $a = -\omega^2 A$

If $t = \frac{T}{4}$, then $a = 0$

If $t = \frac{T}{2}$, then $a = +\omega^2 A$

We can see that only graph (c) will satisfy the above results.



Thus, the correct *a*-*t* graph is represented in option (c).

26 (*d*) From the given graph, we have

Given, $T = 8$s,

Angular frequency, $\omega = \frac{2\pi}{T} = \frac{2\pi}{8} = \left(\frac{\pi}{4}\right) \text{rads}^{-1}$

Equation of motion for the particle executing SHM is given as

$$x = A\sin\omega t$$

$$\therefore \text{ Acceleration, } a = \frac{d^2x}{dt^2} = -\omega^2 x = -\left(\frac{\pi^2}{16}\right)\sin\left(\frac{\pi}{4}t\right)$$

$$(\because A = 1)$$

On substituting $t = \frac{4}{3}$ s, we get

$$a = -\frac{\sqrt{3}}{32}\pi^2 \text{ cms}^{-2}$$

27 (*b*) The acceleration of particle/body executing SHM at any instant (at position *x*) is given as $a = -\omega^2 x$, where ω is the angular frequency of the body.

$$\Rightarrow \quad |a| = \omega^2 x \qquad \ldots\text{(i)}$$

Given, $\qquad x = 5\text{m}$ and $|a| = 20\text{ms}^{-2}$

Substituting the given values in Eq. (i), we get

$$20 = \omega^2 \times 5$$

$$\Rightarrow \quad \omega^2 = \frac{20}{5} = 4 \quad \text{or} \quad \omega = 2 \text{ rad s}^{-1}$$

As we know that, time period, $T = \frac{2\pi}{\omega}$ $\qquad \ldots$(ii)

$\therefore$ Substituting the value of ω in Eq. (ii), we get

$$T = \frac{2\pi}{2} = \pi \text{ s}$$

28 (*d*) For a particle executing SHM, we have maximum acceleration,

$$\alpha = A\omega^2 \qquad \ldots\text{(i)}$$

where, *A* is maximum amplitude and ω is angular velocity of a particle.

Maximum velocity, $\quad \beta = A\omega \qquad$ ...(ii)

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{\alpha}{\beta} = \frac{A\omega^2}{A\omega} \Rightarrow \omega = \frac{\alpha}{\beta}$$

$$\text{But} \quad \omega = \frac{2\pi}{T}$$

$$\therefore \quad \frac{2\pi}{T} = \frac{\alpha}{\beta} \Rightarrow T = \frac{2\pi\alpha}{\beta}$$

Thus, its time period of vibration, $T = \frac{2\pi\beta}{\alpha}$.

29 (*c*) Magnitude of velocity of particle when it is at displacement *x* from mean position, *v*

$$= \omega\sqrt{A^2 - x^2}$$

Also, magnitude of acceleration of particle in SHM,

$$a = \omega^2 x$$

Given, when $x = 2$ cm

$$v = a$$

$$\Rightarrow \omega\sqrt{A^2 - x^2} = \omega^2 x$$

$$\Rightarrow \quad \omega = \frac{\sqrt{A^2 - x^2}}{x} = \frac{\sqrt{9-4}}{2} \qquad [\because \text{given, } A = 3 \text{ cm}]$$

$$\Rightarrow \text{Angular velocity, } \omega = \frac{\sqrt{5}}{2}$$

$$\therefore \text{ Time period of motion, } T = \frac{2\pi}{\omega} = \frac{4\pi}{\sqrt{5}} \text{ s}$$

33 (*d*) Given, $m = 1$kg

The given equation of SHM is

$$x = 6.0\cos\left(100t + \frac{\pi}{4}\right)$$

Comparing it with general equation of SHM,

$$x = A\cos(\omega t + \phi),$$

We have, $\quad A = 6.0 \text{ cm} = \frac{6}{100}$ m and $\omega = 100$ rad/s

Maximum kinetic energy $= \frac{1}{2}m(v_{max})^2$

$$= \frac{1}{2}m(A\omega)^2 = \frac{1}{2}\times 1\times\left[\frac{6}{100}\times 100\right]^2 = 18\text{ J}$$

34 (*d*) Potential energy of an object executing SHM is given by

$$U(x) = \frac{1}{2}kx^2$$

Given, $x = A\cos\omega t$

$\Rightarrow$ $U(x) = \frac{1}{2}kA^2\cos^2\omega t$

At $t = \left(\frac{T}{4}\right)$,

$$U(x) = \frac{1}{2}k^2A^2\cos^2\left(\frac{2\pi}{T}\times\frac{T}{4}\right) \quad \left(\because \omega = \frac{2\pi}{T}\right)$$

$$= \frac{1}{2}k^2A^2\cos^2\left(\frac{\pi}{2}\right) = 0$$

So, PE at $t = T/4$ is zero.

35 (*c*) For SHM,

Displacement, $x(t) = A\cos(\omega t + \phi)$ ...(i)

$\Rightarrow$ $T_2 = 2\pi/\omega$

Potential energy, PE $= \frac{1}{2}kx^2 = \frac{1}{2}kA^2\cos^2(\omega t + \phi)$ ...(ii)

$\Rightarrow$ $= \frac{1}{2}kA^2\frac{[1+\cos 2(\omega t+\phi)]}{2}$

$\therefore$ $T_1 = \frac{2\pi}{2\omega} = \frac{\pi}{\omega}$

$\Rightarrow$ $T_1 = \frac{T_2}{2} \Rightarrow 2T_1 = T_2$

So, the relation between T_1 and T_2 is $2T_1 = T_2$.

36 (*a*) Potential energy of particle in SHM,

$$U = \frac{1}{2}m\omega^2x^2$$

$\Rightarrow$ $U = \frac{1}{2}m(2\pi\nu)^2x^2 = 2\pi^2m\nu^2x^2$

Kinetic energy of particle in SHM

$$K = \frac{1}{2}m\omega^2(A^2 - x^2)$$

$\Rightarrow$ $K = 2\pi^2m\nu^2(A^2 - x^2)$

Hence, total energy

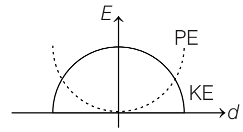
$$E = K + U = 2\pi^2m\nu^2x^2 + 2\pi^2m\nu^2(A^2 - x^2)$$

$$= 2\pi^2m\nu^2A^2 = \frac{2\pi^2mA^2}{T^2} \quad \left(\because T = \frac{1}{\nu}\right)$$

So, total energy depends on amplitude and time period.

37 (*b*) For a particle executing SHM, during oscillation, its KE is maximum at mean position, where PE is minimum. At extreme position, KE is minimum and PE is maximum.

Thus, correct graph is depicted in option (b).



38 (*d*) If a force acting on an object is a function of position only, it is said to be conservative force and it can be represented by a potential energy U function which for one-dimensional case satisfies the derivative condition, i.e.

$$F(x) = -\frac{dU}{dx}$$

Given, $U(x) = k[1 - \exp(-x^2)]$

From this we can say that, change in potential energy (ΔU) will depend on initial and final positions only, so particle is under influence of conservative force.

$\therefore$ $F = -\frac{dU}{dx} = -2kx\exp(-x^2)$

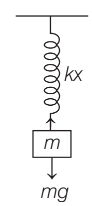
Here, negative sign implies that for any finite non-zero value of x, the force is directed towards the origin. This means there is a restoring force acting on the particle. Hence, the motion of the particle is simple harmonic.

At, $x = 0, F = 0.$

Hence, at equilibrium position force exerted on particle is zero.

Also, potential energy of the particle is minimum at $x = 0$ and at $x = \pm\infty$, the potential energy is maximum.

39 (*c*) At equilibrium the FBD of the block can be shown as



$\Rightarrow$ $mg = kx$

$\Rightarrow$ Stretch, $x = mg/k$

Now, with the extra $\frac{mg}{k}$ stretch, the net stretch become

$$= \frac{mg}{k} + \frac{mg}{k} = \frac{2mg}{k}.$$

40 (*d*) Total energy of the system in SHM $= \frac{1}{2}kA^2$ ...(i)

Kinetic energy of the system in SHM $= \frac{1}{2}mv^2$ ...(ii)

Since, at equilibrium position, i.e. at $x = 0$, the energy of the system would be kinetic only.

$\therefore$ From Eqs. (i) and (ii), we get

$$\frac{1}{2}kA^2 = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\left(\frac{k}{m}\right)A^2} = A\sqrt{\frac{k}{m}}$$

41 (*d*) Let x_1 and x_2 be the extensions in the spring with spring constants k_1 and k_2, respectively.

Then, $x_1 + x_2 = A$ ...(i)

and $k_1x_1 = k_2x_2$ or $\frac{x_1}{x_2} = \frac{k_2}{k_1}$ ...(ii)

From Eq. (ii) substitute the value of x_2 in Eq. (i), we get

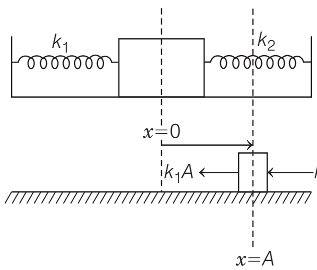
$$x_1 + \frac{k_1}{k_2}x_1 = A \qquad \text{...(iii)}$$

$$k_2x_1 + k_1x_1 = Ak_2$$

On solving these equations, we get

$$x_1 = \left(\frac{k_2}{k_1 + k_2}\right)A$$

42 (c) The situation can be depicted as



As, restoring force of a spring, $F = -kx$

From FBD, $\quad \mathbf{F}_{net} = -(k_1 + k_2)A$

The magnitude of net force, $|\mathbf{F}_{net}| = (k_1 + k_2)A$

43 (b) Kinetic energy (KE) of the system in SHM $= \frac{1}{2}mv^2$

Potential energy (PE) of the system in SHM $= \frac{1}{2}kx^2$

Total kinetic energy of the system

$$= \text{KE} + \text{PE} = \frac{1}{2}kA^2 \qquad \text{...(i)}$$

If KE = PE, then Eq. (i) can be written as,

$$\frac{1}{2}kA^2 = 2 \times \text{PE} = 2\left(\frac{1}{2}kx^2\right)$$

or $$\frac{1}{2}kx^2 = \frac{1}{2}\left[\frac{1}{2}kA^2\right] \Rightarrow x = A/\sqrt{2}$$

44 (b) For a block executing SHM, its

Velocity, $v = -A\omega\sin(\omega t + \phi)$ ...(i)

$$\Rightarrow \quad T_1 = \frac{2\pi}{\omega}$$

$$\text{KE} = \frac{1}{2}mv^2 = \frac{1}{2}mA^2\omega^2\sin^2(\omega t + \phi)$$

$$= mA^2\omega^2[1 - \cos(2\omega + 2\phi)] \Rightarrow T_2 = \frac{2\pi}{2\omega} \quad \text{...(ii)}$$

$$\therefore \quad T_2 = \frac{T_1}{2} \quad \text{So, } T_1 = 2T_2$$

45 (c) Frequency of oscillation of a spring mass system is

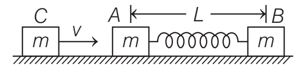
given as $$\nu = 2\pi\sqrt{\frac{m}{k}}$$

This means, it is independent of acceleration due to gravity.

Given, $m = 2\text{kg}$, $k = 10\ \text{Nm}^{-1}$

$$\Rightarrow \quad \nu = 2\pi\sqrt{\frac{2}{10}} = 2\pi\sqrt{0.2} = 2.8\ \text{Hz}$$

46 (a) Given condition can be seen in the figure given below



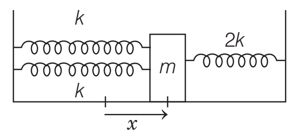
If collision is elastic, C will stop and A will start moving with speed v towards B. At maximum compression (say x), both A and B will move with same speed $v/2$.

$\therefore$ At maximum compression, applying the conservation of energy, we get

$$\frac{1}{2}mv^2 = \frac{1}{2}(2m)\left(\frac{v}{2}\right)^2 + \frac{1}{2}kx^2$$

$$\Rightarrow \quad x = v\sqrt{\frac{m}{2k}}$$

47 (c) Since, the three springs with spring constants k, k and $2k$ are attached to the mass m in parallel. So, net spring constant $= k + k + 2k = 4k$



$\therefore$ The net restoring force on mass m,

$$|\mathbf{F}_{net}| = -4kx \qquad \text{...(i)}$$

As, force in SHM, $|\mathbf{F}_{SHM}| = -m\omega^2 x$ ...(ii)

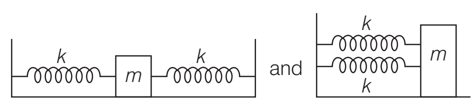
On comparing Eqs. (i) and (ii), we get

$$m\omega^2 = 4k$$

$$\omega = \sqrt{\frac{4k}{m}}$$

$$\therefore \quad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{4k}}$$

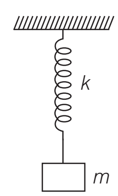
48 (a) For systems,



there are two springs of same spring constants k, attached to mass m in parallel. So, both systems have same time periods.

So, the correct option is (a).

49 (d) As we know that, time period for a spring-mass system as shown



$$T = 2\pi\sqrt{\frac{m}{k}}$$

Case I $\quad T_1 = 2\pi\sqrt{\frac{m}{k}} = 3\ \text{s} \qquad$ ...(i)

Case II When the mass m is increased by 1 kg

$$T_2 = 2\pi\sqrt{\frac{m+1}{k}} = 5\ \text{s} \qquad \text{...(ii)}$$

Dividing Eq. (ii) by Eq. (i), we get

$$\frac{T_2}{T_1} = \sqrt{\frac{m+1}{m}}$$

$$\Rightarrow \quad \frac{5}{3} = \sqrt{\frac{m+1}{m}}$$

$$\Rightarrow \quad \frac{25}{9} = \frac{m+1}{m}$$

$$\Rightarrow \quad \frac{25}{9} = 1 + \frac{1}{m}$$

$$\Rightarrow \quad \frac{1}{m} = \frac{16}{9}$$

$$\therefore \quad m = \frac{9}{16}\text{kg}$$

50 (d) Since, the given spring with spring constants k_1 and k_2 are in parallel, so the net spring constant of the system is $k_{\text{net}} = k_1 + k_2$.

Initially frequency of oscillation,

$$f = \frac{1}{2\pi}\sqrt{\frac{k_{\text{net}}}{m}} = \frac{1}{2\pi}\sqrt{\frac{k_1 + k_2}{m}} \quad \ldots\text{(i)}$$

Now, when k_1 and k_2 are made four times their original value, then

$$f' = \frac{1}{2\pi}\sqrt{\frac{k'_1 + k'_2}{m}} = \frac{1}{2\pi}\sqrt{\frac{4k_1 + 4k_2}{m}} = 2f$$

[$\because$ using Eq. (i)]

Thus, the frequency of oscillation becomes $2f$.

51 (b) Given, $m_1 = 1\text{kg}$,

Extension in length, $l_1 = 5 \text{ cm} = 5 \times 10^{-2}$ m

$$\therefore \quad m_1 g = k l_1$$

where, k = spring constant of the spring

$$\Rightarrow \quad k = \frac{m_1 g}{l_1} = \frac{1 \times 10}{5 \times 10^{-2}} = 200 \text{ Nm}^{-1}$$

So, the spring constant of the given spring is 200 Nm^{-1}.

Now, if a 2 kg block is suspended to this spring and pulled, then

Time period of the block,

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{2}{200}} = 2\pi \times \frac{1}{10} = \frac{\pi}{5}\text{ s}$$

As, maximum velocity, $v_{\max} = A\omega$

where, A = amplitude = 10 cm = 10×10^{-2} m (given)

$$\Rightarrow \quad v_{\max} = A \times \frac{2\pi}{T} = 10 \times 10^{-2} \times \frac{2\pi}{\pi/5}$$

$$= 10^{-1} \times 2 \times 5 = 1 \text{ ms}^{-1}$$

52 (d) The time period T of a simple pendulum of length l is given by

$$T = 2\pi\sqrt{\frac{l}{g}} = \frac{1}{\text{frequency } (n)}$$

where, g is acceleration due to gravity.

$$\therefore \quad \frac{n_1}{n_2} = \sqrt{\frac{l_2}{l_1}} \Rightarrow \frac{l_1}{l_2} = \left(\frac{n_2}{n_1}\right)^2$$

Given, $\frac{n_1}{n_2} = \frac{2}{3}$ or $\frac{n_2}{n_1} = \frac{3}{2}$

$$\Rightarrow \quad \frac{l_1}{l_2} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

53 (d) The distance s covered by the mass falling from height h during its time of fall t is given by

$$s = h = ut + \frac{1}{2}gt^2$$

As, $$u = 0 \Rightarrow h = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2h}{g}} \quad \ldots\text{(i)}$$

The time period of simple pendulum is

$$T = 2\pi\sqrt{\frac{l}{g}} \quad \ldots\text{(ii)}$$

where, l is the length of the pendulum.

From Eqs. (i) and (ii), since h and l are constants, so we can conclude that

$$t \propto \frac{1}{\sqrt{g}} \quad \text{and} \quad T \propto \frac{1}{\sqrt{g}}$$

$$\therefore \quad \frac{t}{T} = 1$$

Thus, the ratio of time of fall and time period of pendulum is independent of value of gravity g or any other parameters like mass and radius of the planet. Thus, the relation between t' and T' on another planet irrespective of its mass or radius will remains same as it was on earth, i.e. $t' = 2T'$.

54 (b) Time period of a simple pendulum which is suspended to the ceiling of the lift, which is initially at rest is given by

$$T = 2\pi\sqrt{\frac{l}{g}} \quad \ldots\text{(i)}$$

When the lift is moving up with an acceleration a, then time period becomes

$$T' = 2\pi\sqrt{\frac{l}{g+a}}$$

Given, $$T' = \frac{T}{2}$$

$$\Rightarrow \quad T' = \frac{T}{2} = 2\pi\sqrt{\frac{l}{g+a}} \quad \ldots\text{(ii)}$$

On dividing Eq. (ii) by Eq. (i), we get

$$\frac{1}{2} = \sqrt{\frac{g}{g+a}}$$

$$\Rightarrow \quad g + a = 4g$$

$$\Rightarrow \quad a = 3g$$

55 (*b*) As we know that, according to force of gravitation, at the surface of the earth for a simple pendulum of mass m

$$mg = \frac{GMm}{R^2} \quad \ldots\text{(i)}$$

where, M is the mass of earth.

When it is taken to a height h, above the earth's surface, then

$$mg' = \frac{GMm}{(h+R)^2} \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$g' = g\left(1+\frac{h}{R}\right)^{-2}$$

$$= g\left(1+\frac{2R}{R}\right)^{-2} \quad (\text{given, } h = 2R)$$

$$= g\,(3)^{-2}$$

The time period of a simple pendulum is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$\therefore$ Ratio of time period T_1 of a simple pendulum, when on the earth's surface and T_2 when on height $2R$ above the earth's surface is $\dfrac{T_1}{T_2} = \sqrt{\dfrac{g'}{g}}$

$$\therefore \quad \frac{T_1}{T_2} = \sqrt{\frac{g(3)^{-2}}{g}} = \sqrt{\frac{1}{3^2}}$$

$$\Rightarrow \quad T_2 = 3T_1 \Rightarrow \frac{T_1}{T_2} = \frac{1}{3}$$

56 (*d*) Here, the rod is oscillating about an end point O. Hence, moment of inertia of rod about the point of oscillation is $I = \dfrac{1}{3}ml_0^2$

Moreover, length l of the pendulum = distance from the oscillation axis to centre of mass of rod $= l_0/2$

$\therefore$ Time period of oscillation,

$$T = 2\pi\sqrt{\frac{I}{mgl}} = 2\pi\sqrt{\frac{\frac{1}{3}ml_0^2}{mg\left(\frac{l_0}{2}\right)}}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{2l_0}{3g}}$$

57 (*d*) We know that,

Time period of a pendulum is given by

$$T = 2\pi\sqrt{L/g_{\text{eff}}} \quad \ldots\text{(i)}$$

Here, L is the length of the pendulum and g_{eff} is the effective acceleration due to gravity in the respective medium in which bob is oscillating.

Initially, when bob is oscillating in air, $g_{\text{eff}} = g$.

So, initial time period, $T = 2\pi\sqrt{\dfrac{L}{g}}$ …(ii)

Let ρ_{bob} be the density of the bob.

When this bob is dipped into a liquid whose density is given as

$$\rho_{\text{liquid}} = \frac{\rho_{\text{bob}}}{16} = \frac{\rho}{16} \quad \text{(given)}$$

$\therefore$ Net force on the bob is

$$F_{\text{net}} = V\rho g - V\cdot\frac{\rho}{16}\cdot g \quad \ldots\text{(iii)}$$

(where, V = volume of the bob = volume of displaced liquid by the bob when immersed in it). If effective value of gravitational acceleration on the bob in this liquid is g_{eff}, then net force on the bob can also be written as

$$F_{\text{net}} = V\rho g_{\text{eff}} \quad \ldots\text{(iv)}$$

Equating Eqs. (iii) and (iv), we have

$$V\rho g_{\text{eff}} = V\rho g - V\rho g/16$$

$$\Rightarrow \quad g_{\text{eff}} = g - g/16 = \frac{15}{16}g \quad \ldots\text{(v)}$$

Substituting the value of g_{eff} from Eq. (v) in Eq. (i), the new time period of the bob will be

$$T' = 2\pi\sqrt{\frac{L}{g_{\text{eff}}}} = 2\pi\sqrt{\frac{16}{15}\frac{L}{g}}$$

$$\Rightarrow \quad T' = \sqrt{\frac{16}{15}} \times 2\pi\sqrt{\frac{L}{g}}$$

$$= \frac{4}{\sqrt{15}} \times T \quad \text{[using Eq. (ii)]}$$

59 (*b*) For this case, $\omega_d b << m(\omega^2 - \omega_d^2)$,

So, from amplitude in the case of forced oscillations

$$A = \frac{F_0}{\{m^2(\omega^2-\omega_d^2)^2 + \omega_d^2 b^2\}^{1/2}}$$

we get $\quad A = \dfrac{F_0}{m(\omega^2 - \omega_d^2)}$

60 (*b*) The amplitude of forced oscillation,

$$A = \frac{F_0}{\{m^2(\omega^2-\omega_d^2)^2 + \omega_d^2 b^2\}^{1/2}}$$

when driving frequency ω_d is close to natural frequency ω, so we can take $(\omega_d = \omega)$.

Hence, $A = \dfrac{F_0}{\omega_d b}$.

61 (*c*) Amplitude of a damped oscillator is given as

$$A = A_0\, e^{-kt}$$

where, $\quad k = \dfrac{-b}{2m}$.

When the amplitude decreases to 0.9 times in 5s, then

$$0.9\,A_0 = A_0 e^{-k5}$$

$$0.9 = e^{-k5}$$

$$\ln(0.9) = -k5 \quad \dots \text{(i)}$$

In another 10 s, the decrease in amplitude is α-times, then

$$\alpha A_0 = A_0 e^{-k(15)} \text{ or } \alpha = e^{-k(15)}$$

$$\Rightarrow \quad \ln(\alpha) = -k(15) = -k(5)(3)$$

Using Eq. (i), we can write

$$\ln(\alpha) = +\ln(0.9)(3)$$

$$\Rightarrow \quad \ln(\alpha) = \ln(0.9)^3$$

$$\alpha = (0.9)^3 = 0.729$$

62 (*a*) Given, frequency of oscillations is

$$f = 5 \text{osc s}^{-1}$$

$\Rightarrow$ Time period of oscillations is $T = \frac{1}{f} = \frac{1}{5}$s

So, time for 10 oscillations is $= \frac{10}{5} = 2$ s

Now, if A_0 = initial amplitude at $t = 0$ and γ = damping factor, then for damped oscillations, amplitude after t second is given as

$$A = A_0 e^{-\gamma t}$$

$\therefore$ After 2 s,

$$\frac{A_0}{2} = A_0 e^{-\gamma(2)} \Rightarrow 2 = e^{2\gamma}$$

$$\Rightarrow \quad \gamma = \frac{\log 2}{2} \quad \dots \text{(i)}$$

Now, when amplitude is $\frac{1}{1000}$ of initial amplitude, i.e.

$$\frac{A_0}{1000} = A_0 e^{-\gamma t}$$

$$\Rightarrow \quad \log(1000) = \gamma t$$

$$\Rightarrow \quad \log(10^3) = \gamma t$$

$$3\log 10 = \gamma t$$

$$\Rightarrow \quad t = \frac{2 \times 3\log 10}{\log 2} \quad \text{[using Eq. (i)]}$$

$$\Rightarrow \quad t = 19.93 \text{ s} \quad \text{or} \quad t \approx 20 \text{ s}$$

63 (*d*) If the body is given a small displacement from the position, a force comes into play which tries to bring the body back to the equilibrium point, giving rise to oscillations or vibrations.

There is no significant difference between oscillations and vibrations. It seems that, when frequency is small, then it is called oscillations (like, the oscillation of branch of a tree), while when frequency is high, then it is called vibration (like, the vibration of a string of musical instrument).

Thus, vibrations and oscillations are not two different types of motion.

Therefore, Assertion is incorrect but Reason is correct.

64 (*d*) $x(t) = A \sin \omega t$ is a sinusoidal periodic function that can represent an oscillatory motion.

Also, $\sin\theta$ is a sinusoidal periodic function.

Therefore, Assertion is incorrect but Reason is correct.

65 (*b*) $x = A \cos \omega t$ and $x = A \sin \omega t$ both represent the displacement of particle undergoing periodic motion.

At $t = 0$,

If $x = A$, we can represents its displacement by

$$x = A \cos \omega t$$

and if $x = 0$, we can represent its displacement by

$$x = A \sin \omega t$$

This implies, both $x = A \cos \omega t$ and $x = A \sin \omega t$, represents the same motion depending on initial position of particle.

Since, $x = A \cos \omega t$ represents a periodic function with time period of 2π rad. So, if the argument of this function ωt is increased by an integral multiple of 2π rad, the value of the function remains the same.

Therefore, Assertion and Reason are correct but Reason is not the correct explanation of Assertion.

66 (*c*) In $x = A \cos \omega t$, since $\cos \omega t$ varies between $+1$ to -1, thus the value of x varies between $+A$ and $-A$.

Amplitude A is a scalar quantity.

Therefore, Assertion is correct but Reason is incorrect.

67 (*b*) In oscillatory or vibratory motion, an object moves about an equilibrium position due to a restoring force.

When the body is at equilibrium position, no net external force acts on it, i.e. $F_{net} = 0$. Therefore, if it is left there at rest, it remains there forever.

If the body is then given then a small displacement from that position, a force comes into play, i.e. restoring force which tries to bring the body back to the equilibrium point giving rise to oscillation or vibrations.

Therefore, Assertion and Reason are correct but Reason is not the correct explanation of Assertion.

68 (*c*) Time period of oscillation of spring mass system is

$$T = 2\pi\sqrt{\frac{m}{k}}$$

which is independent of the amplitude.

Thus, if the amplitude of the system is increased, then T will remain same.

Therefore, Assertion is correct but Reason is incorrect.

69 (*a*) A stiff spring has large spring constant k and a soft spring has small k.

As, frequency of oscillation, for a spring mass system,

$$\nu = 2\pi\omega = 2\pi\sqrt{\frac{k}{m}} \quad \dots \text{(i)}$$

This means, a block of mass m attached to stiff spring have large frequency of oscillation according to Eq. (i).

Therefore, Assertion and Reason are correct and Reason is the correct explanation of Assertion.

70 (*d*) Time period of simple pendulum when it is falling under acceleration of a is

$$T = 2\pi\sqrt{\frac{l}{g-a}} \quad \ldots\text{(i)}$$

In the case of freely falling, $a = g$.

$\therefore$ From Eq. (i),

$$T = 2\pi\sqrt{\frac{l}{g-g}}$$

$$\Rightarrow \quad T = \infty$$

Therefore, Assertion is incorrect but Reason is correct.

71 (*d*) In damped oscillations, the energy of the system is dissipated continuously although the motion is approximately periodic for small damping but not strictly periodic.

This is because, due to the presence of dissipative forces, such as drag friction, etc. the amplitude of oscillation decreases.

Thus, Assertion is incorrect but Reason is correct.

72 (*d*) Total mechanical energy of an oscillation executing SHM, $E = \frac{1}{2}kA^2$

But for damped oscillation, $A(t) = Ae^{-bt/2m}$

So, for damped oscillation,

$$E = \frac{1}{2}k\,(Ae^{-bt/2m})^2 = \frac{1}{2}kA^2\,e^{-bt/m}$$

Thus, E decreases with time t.

Therefore, Assertion is incorrect but Reason is correct.

73 (*c*) Air drag and friction at the support oppose the motion of the pendulum and dissipate its energy gradually. Thus, the pendulum is said to be executing damped oscillations.

However, for small damping, the oscillations remain approximately periodic.

Therefore, Assertion is correct but Reason is incorrect.

74 (*d*) When a system is displaced from its equilibrium position and released, it oscillates with its natural frequency ω and the oscillations are called free oscillations.

All free oscillations eventually die out because of the ever present damping forces. However, an external agency can maintain these oscillations. These are called forced or driven oscillations.

The most familiar example of forced oscillation is when a child in a garden swing periodically presses his feet against the ground or someone else periodically gives the child a push to maintain the oscillations.

Therefore, Assertion is incorrect but Reason is correct.

75 (*d*) For forced oscillations, external force can be represented as

$$F_{\text{ext}} = F_0 \cos \omega_d t$$

This means, F_{ext} varies with time and is not constant. Also, this force helps in sustaining the oscillations.

Thus, these types of oscillations are called forced or driven oscillations.

Thus, Assertion is incorrect but Reason is correct.

76 (*d*) Resonance is a phenomenon of increase in amplitude, when driving frequency is equal to natural frequency of the system.

At resonance amplitude is maximum, but it can never be infinity due to the ever present dissipative forces in nature.

Therefore, Assertion is incorrect but Reason is correct.

77 (*a*) The army troops are suggested to break their march because the hanging bridge could collapse, if the frequency of their march and the frequency at which bridge oscillate would match.

As, it this point the condition of the resonance would be satisfied. Thus, the amplitude with which the bridge was oscillating would increase, thereby leading the bridge to collapse.

Therefore, Assertion and Reason are correct and Reason is the correct explanation of Assertion.

79 (*c*) We know that, for a spring mass system, restoring force, $\mathbf{F}_s = -k\,\mathbf{x}$ (spring force) ...(i)

where, k is spring constant.

$$\mathbf{F} = -m\omega^2\,\mathbf{x} \text{ (for SHM condition)} \quad \ldots\text{(ii)}$$

On comparing Eqs. (i) and (ii), we get

$$\omega = \sqrt{\frac{k}{m}} \Rightarrow T = 2\pi\sqrt{\frac{m}{k}} \qquad \left(\because T = \frac{2\pi}{\omega}\right)$$

$$\Rightarrow \quad T \propto \sqrt{m} \quad \Rightarrow T \propto \frac{1}{\sqrt{k}}$$

So, time period T does not depends on the amplitude of the oscillation

but depend on m and k.

So, statements II and III are correct but I is incorrect.

80 (*a*) Let simple harmonic motions be represented by

$$y_1 = a\sin\left(\omega t - \frac{\pi}{4}\right);\ y_2 = a\sin\omega t$$

and $y_3 = a\sin\left(\omega t + \frac{\pi}{4}\right)$.

On superimposing, resultant SHM will be

$$y = a\left[\sin\left(\omega t - \frac{\pi}{4}\right) + \sin\omega t + \sin\left(\omega t + \frac{\pi}{4}\right)\right]$$

$$= a\left[2\sin\omega t\cos\frac{\pi}{4} + \sin\omega t\right]$$

$$= a\,[\sqrt{2}\sin\omega t + \sin\omega t] = a\,(1+\sqrt{2})\sin\omega t$$

Thus, this function represents SHM with time period $T = \frac{2\pi}{\omega}$ and resultant amplitude, $A = (1+\sqrt{2})a$.

As, energy in SHM $\propto$ (amplitude)^2

$$\therefore \quad \frac{E_{\text{resultant}}}{E_{\text{single}}} = \left(\frac{A}{a}\right)^2 = (\sqrt{2}+1)^2 = (3+2\sqrt{2})$$

$$\Rightarrow \quad E_{\text{resultant}} = (3+2\sqrt{2})E_{\text{single}}$$

Also, the phase of the resultant motion y relative to the first motion y_1 is differ by $\frac{\pi}{4}$.

Thus, the statement given in option (a) is correct, rest are incorrect.

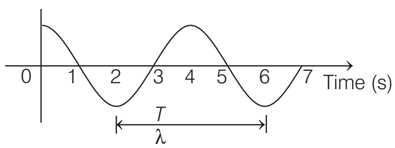
81 (*d*) Actually, the motion of moon callisto is uniform circular motion.

However, what Galileo observed the projection of that uniform circular motion in a line of plane of motion.

Hence, when it was viewed from earth, it looked like a to and fro motion, i.e. a simple harmonic motion.

Thus, the statements given in options (a), (b) and (c) are all correct.

82 (*b*) It is clear from the curve that points corresponding to $t = 2$ s and $t = 6$ s are separated by a distance λ belongs to one time period. Hence, these points must be in same phase.

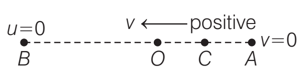


Similarly, points belonging to $t = 0$ s and $t = 2$ s are separated by half the distance that belongs to one time period. Hence, they are not in phase.

However, points belonging to $t = 3$ s and $t = 5$ s or $t = 1$ s and 7 s are at separation of different time period, hence they must not be in phase.

Thus, the statement given in option (b) is correct, rest are incorrect.

83 (*b*) Consider the diagram,



where, the direction from A to B is taken as positive.

(a) When the particle is 3cm away from A going towards B, velocity is towards AB, i.e. positive.

As, in SHM, acceleration and force is always towards mean position O. So in the given case, it will also be positive in this case.

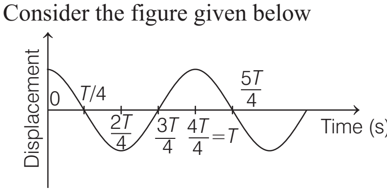
(b) When the particle is at C going towards B, then the velocity is towards B. Hence, it will also be positive.

(c) When the particle is 4 cm away from B going towards A, i.e. velocity is towards BA. Thus, velocity will be negative in this case. Similarly, acceleration and force is also negative.

(d) When the particle is at B acceleration and force are towards BA, i.e. negative.

Thus, the statement given in option (b) is incorrect, rest are correct.

84 (*d*) Consider the figure given below



From this figure, it is clear that

(a) at $t = \frac{3T}{4}$, the displacement of the particle is zero.

Hence, the particle executing SHM will be at mean position, i.e. $x = 0$. So, acceleration is zero and force is also zero.

(b) at $t = \frac{4T}{4} = T$, displacement is maximum, i.e. the particle is at extreme position, so acceleration is maximum.

(c) Similarly, at $t = \frac{T}{4}$, the particle will be at to mean position, so velocity will be maximum at this position.

(d) at $t = \frac{2T}{4} = \frac{T}{2}$, the particle will be at extreme position, so KE $= 0$ and PE = maximum.

Thus, the statement given in option (d) is incorrect, rest are correct.

85 (*c*) Let the equation of a SHM is represented as $x = a \sin \omega t$

Assume, mass of the body $= m$.

(a) Total mechanical energy of the body at any time t is

$$E = \frac{1}{2} m\omega^2 a^2 \qquad \text{...(i)}$$

Kinetic energy at any instant t is

$$K = \frac{1}{2} m v^2 = \frac{1}{2} m \left[\frac{dx}{dt}\right]^2 \qquad \left(\because v = \frac{dx}{dt}\right)$$

$$= \frac{1}{2} m\omega^2 (a^2 - x^2)$$

$$\Rightarrow K_{\max} = \frac{1}{2} m\omega^2 a^2 = E \qquad \text{...(ii)}$$

Thus, total energy per cycle is equal to its maximum KE.

(b) KE at any instant t can also be written as

$$K = \frac{1}{2} m\omega^2 a^2 \cos^2 \omega t$$

$$K_{\text{av}} \text{ for a cycle} = \frac{1}{2} m\omega^2 a^2 [(\cos^2 \omega t)_{\text{av}}]$$

$$\text{For a cycle} = \frac{1}{2} m\omega^2 a^2 \left(\frac{0+1}{2}\right)$$

$$= \frac{1}{4} m\omega^2 a^2 = \frac{K_{max}}{2} \quad \text{[from Eq. (ii)]}$$

(c) Velocity, $v = \frac{dx}{dt} = a\,\omega\cos\omega t$

$$v_{mean} = \frac{v_{max} + v_{min}}{2}$$

$$= \frac{a\omega + (-a\omega)}{2} = 0 \quad \text{(for a complete cycle)}$$

$$\Rightarrow \quad v_{max} \neq \frac{2}{\pi} v_{mean}$$

Thus, the statement given in option (c) is incorrect, rest are correct.

86 (*c*) For a particle executing SHM, its kinetic energy,

$$KE = \frac{1}{2} m\omega^2 (A^2 - x^2)$$

$$\Rightarrow \quad (KE)_{max} = \frac{1}{2} m\omega^2 A^2$$

Potential energy, $PE = (PE)_{max} = \frac{1}{2} m\omega^2 A^2$

Total mechanical energy TE = KE + PE

$$= \frac{1}{2} m\omega^2 A^2$$

$$\Rightarrow \quad TE = (KE)_{max} = (PE)_{max}$$

So, if maximum potential energy becomes double, then both total energy and kinetic energy will also become double.

Thus, the statements given in both options (a) and (b) are correct.

87 (*a*) For a body executing SHM, its kinetic energy,

$$KE(x) = \frac{1}{2} m\omega^2 (A^2 - x^2)$$

where, A is the amplitude.

So, KE at $x = 0$, $KE(0) = \frac{1}{2} m\omega^2 A^2 = (KE)_{max}$

x is maximum at A

So, at $x = A$

$$KE = \frac{1}{2} m\omega^2 (A^2 - A^2) = 0$$

$\therefore$ KE is minimum when x is maximum.

Potential Energy $(PE) = \frac{1}{2} m\omega^2 x^2$

So, PE at $x = 0$, $PE(0) = 0$

So, PE is minimum at $x = 0$.

At $x = A$

$$PE = \frac{1}{2} m^0 \omega^2 A^2 = (PE)_{max}$$

$\therefore$ PE is maximum when x is maximum.

Total Energy $(TE) = \frac{1}{2} kA^2 = \frac{1}{2} m\omega^2 A^2$

This means TE remains constant.

Thus, statement given in option (a) is correct, rest are incorrect.

88 (*d*) Both kinetic and potential energies of a particle, i.e. block in SHM vary between zero and maximum values.

Since, total mechanical (KE + PE) is constant for this system, there will be interconversion of KE and PE during motion.

KE will be maximum at mean position (i.e. $x = 0$) and potential energy will be maximum at extreme (i.e. $x = \pm A$).

Thus, all statements given in options (a), (b) and (c) are correct.

89 (*a*) Kinetic energy and potential energy of a particle executing SHM are periodic with period $\frac{T}{2}$.

Time periods of variation of potential and kinetic energies are same.

However, for such a particle the total mechanical energy, i.e. $U(t) + K(t)$ remains constant always.

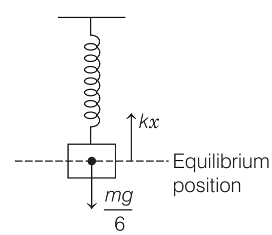
This can also be seen from the graph given in question.

Also, kinetic and potential both energies are positive.

Thus, statement given in option (a) is correct, rest are incorrect.

90 (*b*) If g is the acceleration due to gravity on earth, then acceleration due to gravity on moon is $\frac{g}{6}$.

So, when the spring-mass system is taken to the moon, then the FBD is as shown below.



In equilibrium, $kx = \frac{mg}{6}$

where, k is spring constant and x is the extension in the spring.

$$\Rightarrow \quad x = \frac{mg}{6k}$$

Time period of oscillation for spring-mass system,

$$T = 2\pi \sqrt{\frac{m}{k}}$$

Since, the value of k is governed by the elastic properties of the spring only. Also, mass m remains same everywhere irrespective of its position.

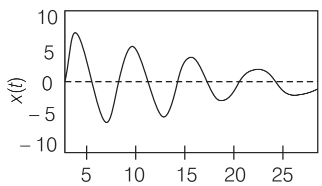
Thus, time period T remains unchanged.

Thus, statement given in option (b) is correct but rest are incorrect.

91 (*b*) The given expression represents the displacement for a damped oscillator.

In this amplitude for a damped oscillation varies as $Ae^{-bt/2m}$.

But this oscillation is approximately periodic, if the damping is small. So, for the given displacement variation w.r.t. time is shown in the graph below.



From this, we can conclude that the amplitude of the damped oscillator decreases with time.

Thus, statement given in option (b) is correct, rest are incorrect.

92 (*c*) From the given figure, amplitude is maximum when $\frac{\omega_d}{\omega} = 1$, i.e. $\omega_d = \omega$.

Also the peak of amplitude is maximum for curve *a* which has least damping. With further increase in damping, amplitude decreases.

Thus, the statements given in options (a) and (b) is correct, so option (c) is correct.

93 (*b*) If a rigid body is moved in such a way that all the particles constituting it undergo circular motion about a common axis, then that type of motion is called rotational motion.

Motion of a pendulum represents to and fro movement about its equilibrium. This represents oscillatory motion.

Motion of car on a straight road represents rectilinear motion.

Motion of a ball thrown by a boy at an angle with horizontal represents projectile motion.

Hence, A $\rightarrow$ 4, B $\rightarrow$ 3, C $\rightarrow$ 2 and D $\rightarrow$ 1.

94 (*b*) For an object executing SHM,

A. $v_{\max} = A\omega$

$$\Rightarrow \quad \frac{v_{\max}}{\text{Amplitude}} = \frac{A\omega}{A} = \omega$$

B. Similarly, $a_{\max} = A\omega^2$

$$\text{So,} \quad \frac{a_{\max}}{\text{Amplitude}} = \frac{A\omega^2}{A} = \omega^2$$

C. If object starts from $x = +A$, its equation is

$$x = A\cos\omega t$$

$$\text{For} \quad x = +A/\sqrt{2}$$

$$\frac{A}{\sqrt{2}} = A\cos\omega t$$

$$\cos\omega t = \frac{1}{\sqrt{2}} = \cos\left(\frac{\pi}{4}\right)$$

$$\Rightarrow \quad \frac{2\pi}{T} \times t = \frac{\pi}{4} \Rightarrow t = T/8$$

D. If object starts from $x = 0$, its equation is

$$x = A\sin\omega t$$

$$\text{For } x = +A/2 \Rightarrow \frac{A}{2} = A\sin\omega t$$

$$\therefore \quad \frac{1}{2} = \sin\omega t$$

$$\Rightarrow \quad \sin\frac{\pi}{6} = \sin\omega t$$

$$\Rightarrow \quad \omega t = \frac{\pi}{6}$$

$$\frac{2\pi}{T}\cdot t = \frac{\pi}{6}$$

$$t = \frac{T}{12}$$

Hence, A $\rightarrow$ 3, B $\rightarrow$ 4, C $\rightarrow$ 1 and D $\rightarrow$ 2.

95 (*c*)

A. Suppose an external force $F(t)$ of amplitude F_0 that varies periodically with time is applied to a damped oscillator (a system representing forced oscillation). Such a force can be represented as

$$F(t) = F_0 \cos\omega_d t \qquad \ldots\text{(i)}$$

where, ω_d = driving frequency.

B. The motion of the particle in such a system is under the combined action of a linear restoring force, damping force and a time dependent driving force is represented by

$$F_{\text{net}} = -kx(t) - bv(t) + F_0\cos\omega_d t$$

$$\Rightarrow m\,a(t) = -kx(t) - b\,v(t) + F_0\cos\omega_d t$$

$$\Rightarrow a(t) = -\frac{k}{m}x(t) - \frac{b}{m}v(t) + \frac{F_0}{m}\cos\omega_d t \qquad \ldots\text{(ii)}$$

C. The oscillator initially oscillates with its natural frequency ω when we apply the external periodic force, the oscillations with the natural frequency die out, and then the body oscillates with the (angular) frequency of the external periodic force. Its displacement after the natural oscillations die out is given by

$$x(t) = A\cos(\omega_d t + \phi)$$

D. Where, amplitude,

$$A = \frac{F_0}{\{m^2(\omega^2 - \omega_d^2)^2 + \omega_d^2 b^2\}^{1/2}}$$

$$\text{and} \qquad \tan\phi = \frac{-v_0}{\omega_d x_0}$$

Here, v_0 and x_0 are velocity and displacement, respectively.

Hence, A $\rightarrow$ 4, B $\rightarrow$ 1, C $\rightarrow$ 2 and D $\rightarrow$ 3.

96 (a) In the given x-t graph, for

(a) No repetition of motion takes place rather it represents a unidirectional, linear but non-uniform motion of the particle, hence motion is non-periodic.

(b) Motion repeats after every 2 s. Hence, it is periodic with time period 2 s.

(c) Motion repeats after every 4s, hence it is periodic with time period of 4 s.

(d) Clearly, the motion repeats itself after 2 s. Hence, periodic having a time-period of 2 s.

97 (a) Given $x(t) = A\cos(\omega t + \phi)$

At $t = 0$; position, $x(t) = 1\,\text{cm}$, velocity, $v = \omega\ \text{cms}^{-1}$

$\Rightarrow$ For $t = 0$, $\quad 1 = A\cos\phi \quad$ ...(i)

Now, $\quad v(t) = \dfrac{dx(t)}{dt} = \dfrac{d}{dt}[A\cos(\omega t + \phi)]$

$$= -A\omega\sin(\omega t + \phi)$$

Again at $t = 0$, $\quad v = \omega\ \text{cms}^{-1} \Rightarrow \omega = -A\omega\sin\phi$

$\Rightarrow \quad -1 = A\sin\phi \quad$ ...(ii)

Squaring and adding Eqs. (i) and (ii), we get

$$A^2\cos^2\phi + A^2\sin^2\phi = (1)^2 + (-1)^2$$

$$A^2 = 2$$

$\Rightarrow \quad A = \pm\sqrt{2}\ \text{cm}$

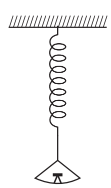
Hence, the amplitude $= \sqrt{2}$ cm

Dividing Eq. (ii) by Eq. (i), we have

$$\frac{A\sin\phi}{A\cos\phi} = \frac{-1}{1} \text{ or } \tan\phi = -1$$

$\Rightarrow$ Initial phase angle, $\phi = -\dfrac{\pi}{4}$

98 (c) As the length of the scale is 20 cm and it can read upto 50 kg. The maximum extension of 20 cm will correspond to maximum weight of $w = mg = 50\ \text{kg} \times 9.8\ \text{ms}^{-2}$.



Using, $\quad \mathbf{F} = -k\mathbf{x}$

$\quad |\mathbf{F}| = F = kx$

As, $\quad F = mg$

$\Rightarrow \quad k = \dfrac{mg}{x}$

Here, substituting the given values, we get

$$k = \frac{50 \times 9.8}{20 \times 10^{-2}} = 2450\ \text{Nm}^{-1}$$

As we know, for a spring mass system time period for oscillation, $T = 2\pi\sqrt{\dfrac{m}{k}}$

or $\quad m = \dfrac{T^2 k}{4\pi^2} = \dfrac{(0.6)^2 \times 2450}{4 \times (3.14)^2} = 22.36\text{kg}$

Weight of the body, $w = mg = 22.36 \times 9.8 = 219.17\text{N}$

$\simeq 220\,\text{N}$

99 (b) Given, spring constant, $k = 1200\ \text{Nm}^{-1}$,

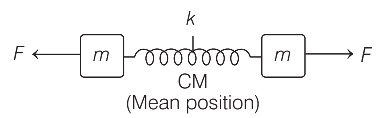
mass, $m = 30\,\text{kg}$

Now, frequency of oscillation,

$$\nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2 \times 3.14}\sqrt{\frac{1200}{3}}$$

$$= \frac{1}{2 \times 3.14} \times 20 = 3.18 \cong 3.2\ \text{s}^{-1}$$

100 (b) The given system of springs can be shown below as



The system is divided into two similar systems with spring divided in two equal halves, $k' = 2k$

Hence, $\quad F = -k'x$

$\Rightarrow \quad F = -2kx$

But $\quad F = ma$

$\therefore \quad ma = -2kx$

$\Rightarrow \quad a = -\left(\dfrac{2k}{m}\right)x \quad$ ...(i)

$\Rightarrow \quad a \propto -x$ (displacement) (as $\dfrac{2k}{m}$ is a constant)

On comparing Eq. (i) with $a = -\omega^2 x$, we get

$$\omega = \sqrt{\frac{2k}{m}}$$

Period of oscillation, $T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{m}{2k}}$

101 (c) Given, angular frequency of the piston,

$\omega = 200\ \text{rad min}^{-1}$

Stroke length = 1 m

$\therefore$ Amplitude of SHM, $A = \dfrac{\text{Stroke length}}{2} = \dfrac{1}{2} = 0.5\ \text{m}$

Now, $\quad v_{\max} = \omega A = 200 \times 0.5 = 100\ \text{m min}^{-1}$

102 (d) On the surface of the earth, time period,

$\therefore \quad T_e = 2\pi\sqrt{\dfrac{l}{g_e}} \quad$ ...(i)

On the surface of the moon, time period,

$$T_m = 2\pi\sqrt{\frac{l}{g_m}} \quad \text{...(ii)}$$

where, g_e and g_m are acceleration due to gravity on the earth and moon surfaces, respectively.

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{T_e}{T_m} = \frac{2\pi}{2\pi}\sqrt{\frac{l}{l} \times \frac{g_m}{g_e}}$$

$\Rightarrow \quad T_m = \sqrt{\dfrac{g_e}{g_m}} \cdot T_e \quad$ ...(iii)

Given, $g_e = 9.8\ \text{ms}^{-2}$, $g_m = 1.7\text{ms}^{-2}$ and $T_e = 3.5$ s

Putting the given values in Eq. (iii), we get

$$T_m = \sqrt{\frac{9.8}{1.7}} \times 3.5 = 8.4 \text{ s}$$

103 (*b*) Mass supported by each wheel = 750 kg

For damping factor *b*, the equation of displacement is

$$x = x_0 e^{-bt/2m}$$

As, $\quad x = x_0/2$

we have $\quad \dfrac{x_0}{2} = x_0 e^{-bt/2m}$

$$\Rightarrow \quad \log_e 2 = \frac{bt}{2m} \text{ or } b = \frac{2m \log_e 2}{t} \quad \ldots\text{(i)}$$

Given, $m = 3000\,\text{kg}$, $g = 9.8\text{ms}^{-2}$ and $x = 15\,\text{cm} = 0.15\,\text{m}$

Restoring force of the system,

$$F = -4kx = mg$$

$$\therefore \quad k = \frac{mg}{4x} = \frac{3000 \times 9.8}{4 \times 0.15}$$

(neglecting negative sign, which is only for direction)

$$\approx 5 \times 10^4\ \text{Nm}^{-1}$$

The time taken in 50% damping, t = One time period = T

$$T = 2\pi\sqrt{\frac{m}{4k}} = 2\pi\sqrt{\frac{3000}{4 \times 5 \times 10^4}} = 0.769 \text{ s}$$

$\therefore$ Substituting values in Eq. (i), we get

$$b = \frac{2 \times 750 \times 0.693}{0.769}$$

$$= 1351.58\ \text{kgs}^{-1} \simeq 1352\,\text{kgs}^{-1}$$

104 (*a*) When displacement is $x = 5\,\text{cm} = 0.05\,\text{m}$

$$\text{Acceleration, } a = -\omega^2 x = -\left(\frac{2\pi}{T}\right)^2 (x)$$

$$= -\left(\frac{2\pi}{0.2}\right)^2 (0.05) = -5\pi^2\ \text{ms}^{-2}$$

$$\text{Velocity, } v = \omega\sqrt{a^2 - x^2} = \left(\frac{2\pi}{T}\right)\sqrt{(0.05)^2 - (0.05)^2}$$

$$= \left(\frac{2\pi}{T}\right) \times 0 = 0$$

105 (*b*) Given, $y = 3\cos\left(\dfrac{\pi}{4} - 2\omega t\right)$

$$\text{Velocity of the particle, } v = \frac{dy}{dt} = \frac{d}{dt}\left[3\cos\left(\frac{\pi}{4} - 2\omega t\right)\right]$$

$$= 6\omega \sin\left(\frac{\pi}{4} - 2\omega t\right)$$

$$\text{Acceleration, } a = \frac{dv}{dt} = \frac{d}{dt}\left[6\omega \sin\left(\frac{\pi}{4} - 2\omega t\right)\right]$$

$$\Rightarrow \quad a = -12\omega^2 y$$

$\Rightarrow$ As acceleration, $a \propto -y$

Hence, motion is SHM.

Clearly, from the equation

$$\omega' = 2\omega$$

[$\because$ Comparing the given equation with standard equation, $y = a\cos(\omega' t + \phi)$]

$$\Rightarrow \quad \frac{2\pi}{T'} = 2\omega \quad \Rightarrow \quad T' = \frac{2\pi}{2\omega} = \frac{\pi}{\omega}$$

So, motion is SHM with period $\dfrac{\pi}{\omega}$.

106 (*b*) Given equation of motion is

$$y = \sin^3 \omega t = (3\sin\omega t - 4\sin 3\omega t)/4$$

$$(\because \sin 3\theta = 3\sin\theta - 4\sin^3\theta)$$

$$\Rightarrow \quad \frac{dy}{dt} = \left[\frac{d}{dt}(3\sin\omega t) - \frac{d}{dt}(4\sin 3\omega t)\right]/4$$

$$\Rightarrow \quad 4\frac{dy}{dt} = 3\omega\cos\omega t - 36 \times [3\omega^2 \cos 3\omega t]$$

$$\Rightarrow \quad 4 \times \frac{d^2y}{dt^2} = -3\omega^2 \sin\omega t + 36\omega^2 \sin 3\omega t$$

$$\Rightarrow \quad \frac{d^2y}{dt^2} = -\frac{3\omega^2 \sin\omega t + 36\omega^2 \sin 3\omega t}{4}$$

$\Rightarrow \quad \dfrac{d^2y}{dt^2}$ is not proportional to y.

Hence, motion is not SHM.

As the expression is involving sine function, hence it will be periodic.

107 (*d*) For a particle executing SHM,

acceleration $a \propto -$ displacement x

which is correctly given in option (d) only.

108 (*d*) According to the question,

displacement, $y = a\sin\omega t + b\cos\omega t \quad \ldots$(i)

Let $\quad a = A\sin\phi$ and $b = A\cos\phi$

Now, $\quad a^2 + b^2 = A^2\sin^2\phi + A^2\cos^2\phi = A^2$

$$\Rightarrow \quad A = \sqrt{a^2 + b^2}$$

Now, substituting the values of a, b and A in Eq. (i), we get

$$\Rightarrow \quad y = A\sin\phi \cdot \sin\omega t + A\cos\phi \cdot \cos\omega t$$

$$= A\sin(\omega t + \phi)$$

[$\because$ using trigonometric identity $\sin(A + B) = \sin A\cos B + \cos A\sin B$]

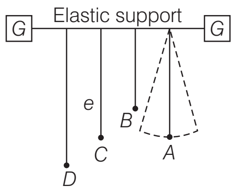
$$\Rightarrow \quad \frac{dy}{dt} = A\omega\cos(\omega t + \phi)$$

$$\Rightarrow \quad \frac{d^2y}{dt^2} = -A\omega^2 \sin(\omega t + \phi) = -Ay\omega^2 = (-A\omega^2)y$$

$$\Rightarrow \quad \frac{d^2y}{dt^2} \propto -y$$

Hence, it is an equation of SHM with amplitude, $A = \sqrt{a^2 + b^2}$.

109 (b) According to the question, A is given a transverse displacement as shown below

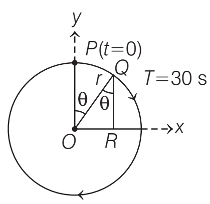


Through the elastic support, the disturbance is transferred to all the pendulums. A and C are having same length, hence they will be in resonance, it is because their time period of oscillation,

$T = 2\pi \sqrt{\dfrac{l}{g}}$ will be same and hence frequency will be same.

So, C will vibrate with maximum amplitude.

110 (a) Let angular velocity of the particle executing circular motion is ω and when it is at Q it makes an angle θ as shown in the diagram.



Clearly, $\theta = \omega t$

Now, we can write $OR = OQ \cos (90° - \theta)$

$$= OQ \sin \theta = OQ \sin \omega t$$

$$= r \sin \omega t \qquad (\because OQ = r)$$

$$\Rightarrow \quad x = r \sin \omega t = B \sin \omega t \qquad (\because r = B)$$

$$= B \sin \frac{2\pi}{T} t = B \sin \left(\frac{2\pi}{30} t\right)$$

Clearly, this equation represents SHM.

111 (c) As the given equation is $x = a \cos (\alpha t)^2$ is a cosine function, hence it is an oscillatory motion.

Now, putting $t + T$ in place of t

$$x(t + T) = a \cos [\alpha (t + T)]^2$$

$$= a \cos (\alpha t^2 + \alpha T^2 + 2\alpha t T) \neq x(t)$$

where, T is supposed to be the period of the given function.

Hence, it is not periodic.

112 (a) Let equation of the particle executing SHM is represented by

$$y = A \sin \omega t$$

Particle's speed, $v = \dfrac{dy}{dt} = A \omega \cos \omega t$

$\Rightarrow$ Maximum speed,

$$(v)_{max} = A\omega = 30 \text{ cms}^{-1} \qquad \text{...(i) (given)}$$

Particle's acceleration, $a = \dfrac{dx^2}{dt^2} = -A\omega^2 \sin \omega t$

Maximum acceleration,

$$|a_{max}| = \omega^2 A = 60 \text{ cms}^{-2} \qquad \text{...(ii) (given)}$$

From Eqs. (i) and (ii), we get

$$\omega (\omega A) = 60$$

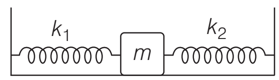
$$\Rightarrow \quad \omega (30) = 60$$

$$\Rightarrow \quad \omega = 2 \text{ rads}^{-1}$$

$$\Rightarrow \quad \frac{2\pi}{T} = 2 \text{ rads}^{-1} \Rightarrow T = \pi \text{ s}$$

$\therefore$ The period of oscillations in π s.

113 (b) In the given figure (as shown below), it can be said that the two springs are connected in parallel.



where, equivalent spring constant $= k_{eq} = k_1 + k_2$.

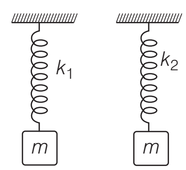
Time period of oscillation of the spring-block system,

$$T = 2\pi \sqrt{\frac{m}{k_{eq}}} = 2\pi \sqrt{\frac{m}{k_1 + k_2}}$$

$$\Rightarrow \quad \nu = \frac{1}{T} = \frac{1}{2\pi} \times \sqrt{\frac{k_1 + k_2}{m}} \qquad \text{...(i)}$$

= equivalent oscillation frequency.

Initially when the mass is connected to the two springs individually as shown below, then



$$\nu_1 = \frac{1}{2\pi} \sqrt{\frac{k_1}{m}} \qquad \text{...(ii)}$$

and

$$\nu_2 = \frac{1}{2\pi} \sqrt{\frac{k_2}{m}} \qquad \text{...(iii)}$$

From Eq. (i), we get

$$\nu = \frac{1}{2\pi} \left[\frac{k_1}{m} + \frac{k_2}{m}\right]^{1/2}$$

$$= \frac{1}{2\pi} \left[\frac{4\pi^2 \nu_1^2}{1} + \frac{4\pi^2 \nu_2^2}{1}\right]^{1/2}$$

$$\left[\begin{array}{l} \because \text{ from Eq. (ii), } \dfrac{k_1}{m} = 4\pi^2 \nu_1^2 \\ \text{and from Eq. (iii), } \dfrac{k_2}{m} = 4\pi^2 \nu_2^2 \end{array}\right]$$

$$= \frac{2\pi}{2\pi} [\nu_1^2 + \nu_2^2]^{1/2}$$

$$\Rightarrow \quad \nu = \sqrt{\nu_1^2 + \nu_2^2}$$